A systematic approach to quality function deployment with a full illustrative example

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Abstract

This paper presents a systematic and operational approach to quality function deployment (QFD), a customer-driven quality management system for product development. After a comprehensive description of the relevant elements in house of quality (HOQ), the first and most influential phase of the QFD system, a 9-step model is proposed to help build such an HOQ. A number of 9-point scales are developed whose uses could help unify the various measurements in HOQ to avoid arbitrariness. Special attention is paid to the various subjective assessments in the HOQ process, and symmetrical triangular fuzzy numbers (STFNs) are suggested for use to capture the vagueness in people’s linguistic assessments. Instead of using the quite subjective sales-point concept, entropy method is introduced to conduct competitive analysis and derive competitive priority ratings. A thorough explanation is given to address the concepts, computations and implementations in the proposed HOQ model, followed by a full example for a fried Chinese vegetable to illustrate step by step all the relevant details with the purpose of facilitating the understanding and application of the QFD process. Two difficult parts omitted from our model, especially the correlation matrices, are discussed in some detail finally, and possible approaches are also suggested to deal with them in a potentially more complete HOQ model.

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1. Introduction

Quality function deployment (QFD) is “a system to assure that customer needs drive the product design and production process” (Ref. [1]). Typically, a QFD system can be broken down into four inter-linked phases to fully deploy the customer needs phase by phase (Refs. [1–5]). In QFD, each phase’s important outputs (HOWs), generated from the phase’s inputs (WHATs), are converted into the next phase as its inputs (new WHATs). So each phase can be described by a matrix of “WHATs” and “HOWs”, which is easy and convenient to deal with in practice. The four QFD phases include: Phase I to translate customer needs into product design attributes which we will call technical measures; Phase II to translate important technical measures into parts characteristics; Phase III to translate important parts characteristics into process operations; and Phase IV to translate key process operations into day to day production requirements.

The first phase of QFD, usually called house of quality (HOQ), is of fundamental and strategic importance in the QFD system, since it is in this phase that the customer needs for the product are identified and then, incorporating the producing company’s competitive priorities, converted into appropriate technical measures to fulfill the needs. In other words, HOQ

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links the “voice of the customer” to the “voice of the technician” through which process and production plans can be
developed in the other phases of the QFD system. The structures and analyzing methods of the other three QFD phases
are essentially the same as the first one, so we will study the HOQ phase only. In fact, most QFD studies focus mainly
on its first phase (for exceptions, see Refs. [1,3,5]).

A house of quality (HOQ) involves the collection and analysis of the “voice of the customer” which includes the
customer needs for a product, customers’ perceptions on the relative importance of these needs and the relative performance
of the producing company and its main competitors on the needs. It also requires the generation and analysis of the “voice
of the technician” which includes the technical measures converted from the customer needs, technicians’ evaluations on
the relationship between each customer need and each technical measure, and the performance of the relevant companies
in terms of these technical measures. With such a large amount of information to be collected and processed, building
an HOQ may be too complex to be complete and comparable. Systematization of the HOQ process is thus a necessity.
Many studies have been done (Refs. [1–9]) and a number of QFD information systems have also been proposed (e.g.,
Refs. [10,11]) towards this purpose. However, most of these works are incomplete in the sense that either they do not
contain all the important elements of HOQ or their quantifications are not very satisfactory.

On the other hand, most information involved in the HOQ process is generated from human beings’ perceptions and
linguistic assessments that are quite subjective and vague. Both the “voice of the customer” and the “voice of the
technician” contain ambiguity and multiplicity of meaning. “Customer need #1 is very important”, “technical measure
#2 has weak relationship with customer need #3” and “company #4 performs well on customer need #5” are examples
of these “voices” which are imprecise in terms of breadth of meaning. Efforts should therefore be made to deal with
the vagueness in these “voices” involved in the HOQ process. Among a few studies in this aspect, Khoo and Ho [12]
provide a fuzzy QFD framework to perform QFD analysis using symmetrical triangular fuzzy numbers (STFNs). However,
they exclude competitive analysis from their framework. Chan et al. [13] also use STFNs to analyze the “voice of the
customer”, but their study does not involve the “voice of the technician”.

This paper presents a systematic and operational approach to the QFD process. After a detailed description and analysis
of the HOQ elements, we provide a 9-step process to build an HOQ. Then, we suggest some feasible methods on how
to collect and analyze the information from both the customers and the technicians. We especially address the various
“voices” in HOQ using symmetrical triangular fuzzy numbers (STFNs) and some of the proposed HOW steps are fuzzified
to produce fuzzy results, which are generally more representative than traditional crisp approaches of using simple numbers.
We also suggest the use of entropy method to perform competitive analyses and obtain competitive priority ratings for both
customer needs and technical measures. A fried Chinese vegetable example is given to fully illustrate our approach. Two
difficult parts omitted from our model—the probability factors for achieving the goals set for the HOWs and especially
the correlation matrices among the WHATs and HOWs—are discussed in some detail finally, and possible approaches
are also suggested to deal with them in a potentially more complete HOQ model.

2. The HOQ process

2.1. The HOQ elements

According to many works (Refs. [1–9,14]), a typical HOQ contains some of the following elements or concepts:
1. Customers: At first the customers of a product or service concerned should be identified by the producing company.
2. Customer needs (WHATs): These are the requirements of customers for the product expressed in customers’
languages.
3. Structuring customer needs: If there are many customer needs, grouping them into meaningful hierarchies or cate-
gegies is necessary for easy understanding and analysis.
4. Correlation matrix of customer needs: This matrix contains the correlation between each pair of customer needs
(WHATs) through empirical comparisons. The information is provided by customers and usually is difficult to obtain
since a lot of pairwise comparisons are needed. The purpose of completing this correlation matrix is for the company to
identify where trade-off decisions and further research may be required. Correlation is usually described by the following
5-point scale (Refs. [2,5,7,9]):

\[
\begin{array}{cccccc}
\text{strong negative} & \text{moderate negative} & \text{no correlation} & \text{moderate positive} & \text{strong positive} \\
1 & 2 & 3 & 4 & 5
\end{array}
\]
5. Relative importance ratings of customer needs: These are the relative importance of customer needs perceived by the customers, usually expressed and measured through a 5-point scale such as:

<table>
<thead>
<tr>
<th></th>
<th>very low</th>
<th>low</th>
<th>moderate importance</th>
<th>high</th>
<th>very high</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

Sometimes the following scale emphasizing more on the right-hand side is used to measure relative importance (Ref. [9]), which was preferred by the Japanese:

<table>
<thead>
<tr>
<th></th>
<th>no (importance)</th>
<th>low (importance)</th>
<th>moderate (importance)</th>
<th>high (importance)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>9</td>
</tr>
</tbody>
</table>

Other scales such as a 9-point scale from 1 to 9 or 100-point scale from 1 to 100 may also be used. For comparison of some scales to measure relative importance, see Ref. [8].

6. Competitors: For the product concerned, the producing company should identify the main competitors in the relevant markets.

7. Customer competitive assessment: This is to let the customers assess the relative performance of the producing company’s product and its main competitors’ similar products on the customer needs identified, usually expressed and measured by the following 5-point scale:

<table>
<thead>
<tr>
<th></th>
<th>very poor</th>
<th>poor</th>
<th>neutral</th>
<th>good</th>
<th>very good</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

8. Goals for customer needs: The producing company can set performance goals on each WHAT to work on in order to better satisfy customer needs. The scale for measuring these goals is the same as (4).

9. Sales-point: A sales-point is a kind of possibility which will give your company a unique business position (Refs. [1,2,6,7]). A “strong” sales point is reserved for important WHATs where each comparing company is rated poorly. A “moderate” sales point means the importance rating or competitive opportunity is not so great. And a “no” sales point means no business opportunity. Numerically, 1.5, 1.25 and 1 are assigned to strong, moderate and no sales point respectively (Refs. [1,7]).

10. Final importance ratings of customer needs: For each WHAT its final importance rating (or row total, planning weight) is calculated by the following formula (Refs. [1,7]):

\[
\text{Final importance rating} = \text{relative importance} \times \text{improvement ratio} \times \text{sales point},
\]

where improvement ratio equals to goal performance level divided by current performance level.

11. Technical measures (HOWs): These are design specifications, substitute quality characteristics, engineering attributes or methods, which can relate to and measure customer needs.

12. Correlation matrix of the HOWs: This matrix is to help the producing company establish which HOWs are correlated and determine the extent of these correlations, which can be obtained through engineering analysis and experience. The scale for measuring correlations among the HOWs is the same as (1).

13. Relationship matrix of WHATs vs. HOWs: This matrix is a systematic means for identifying the level of relationship between each WHAT and each HOW. Usually these relationships are measured by the following scale that, similar to (3), puts more weights on strong relationship (Refs. [2,5,7,9]):

<table>
<thead>
<tr>
<th></th>
<th>no (relation)</th>
<th>weak (relation)</th>
<th>moderate (relation)</th>
<th>strong (relation)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>9</td>
</tr>
</tbody>
</table>

14. Improving directions of the HOWs: It is helpful to identify in which direction each HOW should be improved to better satisfy customer needs. Basically there are three types of improving directions: maximizing (or increasing), minimizing (or decreasing), and meeting targets (or guidelines, standards).

15. Technical competitive assessment: This is to technically evaluate the performance of the company’s product and its main competitors’ similar products on each HOW.

16. Goals for the HOWs: The producing company can set performance goals on each HOW to be more technically competitive.
17. Probability factors: For each HOW’s performance goal there is a probability factor (Refs. [2,9]) to achieve the goal, determined by engineering and cost analysis. The following 5-point scale similar to (2) and (4) is commonly used to measure these probabilities:

<table>
<thead>
<tr>
<th>very low</th>
<th>low</th>
<th>moderate probability</th>
<th>high</th>
<th>very high</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

18. Importance ratings of the HOWs: This is the main output of the HOQ process. Importance rating of a HOW is usually computed using the following simple additive weighting (SAW) formula (Refs. [2,7,9]):

\[
\text{importance rating of a HOW} = \sum (\text{final importance rating of WHAT} \times \text{relationship value between WHAT and the HOW}),
\]

(7)

where the summation is over all WHATs.

2.2. A 9-step HOQ model

In practice, it is both difficult and unnecessary to include all the HOQ elements described above. In fact, different users build different HOQ models involving different elements from the above list. The most simple but widely used HOQ model contains only the customer needs (WHATs) and their relative importance, technical measures (HOWs) and their relationships with the WHATs, and the importance ratings of the HOWs. Some models include further the customer competitive assessment and performance goals for the WHATs. Some authors add one or both of the two correlation matrices into this simple model. Fewer models include the technical competitive assessment since this information is difficult to deal with and, as such, goals and probability factors for the HOWs appear seldom in HOQ studies—even if these are included, they are hardly incorporated into the computation of the importance ratings of the HOWs, which are usually obtained by formula (7) that does not relate to technical competitive assessment at all.

To avoid inconsistencies and facilitate applications, we propose in this paper a unified 9-step HOQ model (Fig. 1) which, a refinement of the model by Chan and Wu [15], contains the frequently used HOQ elements. A noticeable exclusion is the two correlation matrices since these correlations are not easy to obtain, not to say to incorporate into the respective importance ratings (for two possible approaches, see Refs. [12,16]). This, however, does not imply their unimportance in the HOQ process and future effort should be made to handle them properly. Probabilities to achieve goals for the HOWs are not included, either, but they can at least partly be reflected through technical competitive assessment and the improvement ratios. Later in Section 5, we will elaborate more on these two omitted parts and also indicate possible ways to include them into an even more complete HOQ model.

<table>
<thead>
<tr>
<th>Step 1</th>
<th>Step 2</th>
<th>Step 3</th>
<th>Step 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Identify Customers &amp; Collect Customer Needs (WHATs)</td>
<td>Determine Relative Importance Ratings of WHATs</td>
<td>Identify Competitors, Conduct Customer Competitive Analysis &amp; Set Customer Performance Goals for WHATs</td>
<td>Determine Final Importance Ratings of WHATs</td>
</tr>
<tr>
<td>Step 5</td>
<td>Step 6</td>
<td>Step 7</td>
<td>Step 8</td>
</tr>
<tr>
<td>Generate Technical Measures (HOWs)</td>
<td>Determine Relationships Between WHATs and HOWs</td>
<td>Determine Initial Technical Ratings of HOWs</td>
<td>Conduct Technical Competitive Analysis &amp; Set Technical Performance Goals for HOWs</td>
</tr>
<tr>
<td>Step 9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Determine Final Technical Ratings of HOWs</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 1. House of quality (HOQ): a 9-step model.
2.3. Scales and fuzziness

As summarized in Section 2.1, there are many scales used in the HOQ process to measure different concepts. For example, it is common to use scale (1) to measure the correlation between each pair of WHATs or HOWs, scale (6) to measure the relationship between each WHAT and each HOW, scale (2) to measure the relative importance of WHATs, and scale (4) to measure the companies’ current and goal performance in terms of WHATs. But there is not a rule as to which element must or should be measured by a designated scale and, in fact, different users may use different scales to measure the same concept. For example, while many authors adopt scale (2) to measure the importance of WHATs, some people prefer to use scale (3) for the same purpose.

On the other hand, it is well recognized that people’s assessments of concepts are always subjective and thus imprecise, and the linguistic terms people use to express their judgements are vague in nature. Using objective, definite and precise numbers to represent linguistic assessments are, although widely adopted, not very reasonable. A more rational approach is to assign fuzzy numbers to linguistic assessments so that their vagueness can be captured. For example, rather than using numbers 1 and 5 to represent “very low” and “very high” importance in scale (2), we may assign symmetrical triangular fuzzy numbers (STFNs) such as [0.5,1.5] and [4.5,5.5] to these two linguistic assessments to express their vagueness. Here an STFN, in the form of [a, c], is a special fuzzy set representing a fuzzy concept “approximately b” where \( b = (a + c)/2 \) (see Appendix for a brief account of fuzzy set and STFN).

To operate the HOQ process and make relevant computations comparable, it is necessary to unify the various scales used in HOQ. We propose a few 9-point 1-to-9 scales to measure the respective concepts in our proposed HOQ model. These scales can be represented by either conventional numbers or STFNs according to practical requirements. The rationale of adopting such 1-to-9 rating scales is proved by the many tests made by Saaty [17]. On the other hand, since 5-point scale is simple and easy to use and also includes enough information people provide on the attributes measured (Refs. [18–20]), we suggest that for each of the following 9-point scales proposed, a 5-point subscale 1-3-5-7-9 (or corresponding STFNs [0,2]-[2,4]-[4,6]-[6,8]-[8,10]) be used as much as possible. Only when compromises must be made or some uncertainty exists, the numbers 2, 4, 6, 8 (or corresponding STFNs [1,3], [3,5], [5,7], [7,9]) are adopted. These suggested scales are:

- For measuring the relative importance of WHATs:

  \[
  \begin{array}{ccccccc}
  \text{very low} & \text{low} & \text{moderate importance} & \text{high} & \text{very high} \\
  \hline
  1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
  \end{array}
  \]

- For measuring the companies’ current and goal performance in terms of WHATs:

  \[
  \begin{array}{ccccccc}
  \text{very poor} & \text{poor} & \text{neutral} & \text{good} & \text{very good} \\
  \hline
  1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
  \end{array}
  \]

- For measuring the relationship between each WHAT and each HOW:

  \[
  \begin{array}{ccccccc}
  \text{very weak} & \text{weak} & \text{moderate relation} & \text{strong} & \text{very strong} \\
  \hline
  1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
  \end{array}
  \]

3. Descriptions of the HOQ model

According to the above preparations, our proposed 9-step HOQ model (Fig. 1) can be described as follows. These descriptions, both qualitative and quantitative, are based on the ideas from Refs. [1–9,13–15].

**Step 1. Identify customers and collect their needs (WHATs):** The producing company should know who are the customers for the product concerned. There are generally three types of customers, internal customers such as shareholders, managers and employees, intermediate customers such as wholesale people and retailers, and ultimate customers such as recipients of service, purchasers, institutional purchasers. Usually the main focus is on the ultimate customers who could be identified through previous information and marketing research.
Understanding what customers need for a product is important for the company, otherwise you cannot know how to satisfy your customers and thus how to keep your business successful. Available methods to collect customer needs include focus group, individual interviews, listening and watching, and using existing information. It is suitable and economical to gather customer needs through focus group and individual interviews (Ref. [2]). As for how many customers should be interviewed, see Refs. [7,8] for analysis and guidelines.

Grouping related customer needs into a category is helpful in analyzing the needs. Affinity Diagram (Refs. [2,6,7]), a method of arranging random data into natural and logical groups, can be used to organize customer needs. Cluster analysis can also be used for this purpose (Ref. [8]). Usually customer needs can be organized as a tree-like structure with an increasing number of items moving from left/top (higher levels) to right/bottom (lower levels).

Suppose that, through appropriate ways, $K$ customers have been selected and $M$ customer needs have been identified based on the opinions of these $K$ customers. The $M$ customer needs are denoted as $W_1, \ldots, W_M$. These needs could be classified into some meaningful categories according to practical situation.

**Step 2. Determine the relative importance ratings of customer needs:** Customer needs (WHATs) usually are of different degrees of importance and it is a common practice for the company to focus more on the important WHATs. The relative importance of the WHATs is usually expressed as a set of ratings that can be determined by letting the customers reveal their perceptions on the relative importance of the WHATs and then averaging their perceptions. The appropriate ways of obtaining customers’ perceptions are by individual interviews and mail surveys.

Suppose that for customer need $W_m$, customer $k$ supplies a relative importance rating $g_{mk}$ to it according to scale (9), where $g_{mk}$ is one of the nine crisp numbers or STFNs in scale (8). Then the resulting average relative importance rating for $W_m$ is computed by

$$g_m = \frac{(g_{m1} + g_{m2} + \cdots + g_{MK})}{K} = \sum_{k=1}^{K} g_{mk}/K, \quad m = 1, 2, \ldots, M. \quad (11)$$

It should be noted that if STFNs are used in scale (8), then $g_m$ computed by (11) is also an STFN. For convenience, the relative importance ratings of the $M$ customer needs can be described as an $M$-dimensional vector, $g = (g_1, g_2, \ldots, g_M)$.

**Step 3. Identify competitors and conduct customer competitive analysis:** Competitors who produce the similar products should be identified by the company under study. Knowing the company’s strengths and constraints in all aspects of a product and in comparison with its main competitors is essential for a company if it wishes to improve its competitiveness in the relevant markets. This kind of information can be obtained by asking the customers to rate the relative performance of the company and its competitors on each WHAT and then to aggregate the customers’ ratings. Useful ways of conducting this kind of comparison analysis are also via mailed surveys and individual interviews.

Denote the company in question by $C_1$. Suppose that $L - 1$ competitors are identified, denoted as $C_2, \ldots, C_L$. Then the $K$ customers are requested to provide their perceptions on the relative performance of these $L$ companies’ products of the similar type in terms of the $M$ customer needs. Suppose that customer $k$ supplies a rating $x_{mlk}$ on company $C_l$’s performance in terms of $W_m$ using scale (9), where $x_{mlk}$ is one of the nine crisp numbers or STFNs in scale (9). Then the performance rating of company $C_l$ on customer need $W_m$ is given as

$$x_{ml} = \frac{(x_{m1} + x_{m2} + \cdots + x_{ML})}{K} = \sum_{k=1}^{K} x_{mlk}/K, \quad m = 1, 2, \ldots, M, \quad l = 1, 2, \ldots, L. \quad (12)$$

Thus, the companies’ performance ratings on the customer needs can be denoted by an $M \times L$ matrix, called customer comparison matrix:

$$X = \begin{bmatrix}
C_1 & C_2 & \cdots & C_L \\
W_1 & x_{11} & x_{12} & \cdots & x_{1L} \\
W_2 & \vdots & \vdots & \ddots & \vdots \\
\cdots & \vdots & \vdots & \ddots & \vdots \\
W_M & x_{M1} & x_{M2} & \cdots & x_{ML}
\end{bmatrix}_{M \times L}$$

Based on this $X$ information, customer competitive priority ratings on the WHATs for the producing company $C_1$ can be obtained, usually using the quite subjective sales point concept (see Element 9 of Section 2.1), as $e = (e_1, e_2, \ldots, e_m)$ where $e_m$ is company $C_1$’s priority rating on customer need $W_m$. This set of priority ratings can also be derived by the more objective entropy method as introduced in the Appendix.

According to company $C_1$’s current performance on the WHATs in relation to its competitors’ performance, performance goals on the WHATs can be set for the company. These goals should be set competitively and realistically by the company, which is a highly strategical activity involving many considerations from relevant management. Assume that for customer...
need \( W_m \), a proper performance goal \( a_m \) has been set according to scale (9). Thus the company has a goal performance vector in terms of the customer needs, denoted as \( a = (a_1, a_2, \ldots, a_M) \). In most cases, each goal performance level should not be lower than current performance level, implying the need or desire for further improvement. From this we can also set the company’s improvement ratio for \( W_m \) as \( u_m = a_m / x_m \). It is obvious that the higher the improvement ratio, the more the company should work on the WHAT, and thus the more important the WHAT for the company.

**Step 4. Determine the final importance ratings of customer needs:** Customer needs with higher relative importance perceived by customers and higher competitive priorities and improvement ratios should receive higher attention. Thus, according to (5), customer need \( W_n \)’s final importance rating for the company is determined jointly by its relative importance \( g_m \), competitive priority \( e_m \) and improvement ratio \( u_m \) as

\[
f_m = u_m \times g_m \times e_m, \quad m = 1, 2, \ldots, M.
\]  

(13)

WHATs with high such final ratings indicate both importance and potential business benefit to the company.

Of course, a (weighted) sum of \( u_m \), \( g_m \) and \( e_m \) can also produce a reasonable \( f_m \), if preferred. In any case, we will denote the final importance ratings for the customer needs as a vector, \( f = (f_1, f_2, \ldots, f_M) \).

**Step 5. Generate technical measures (HOWs):** After customers reveal their needs for the product, the company’s technicians or product development team should develop a set of HOWs to capture the customer needs in measurable and operable technical terms. HOWs could be generated from current product standards or selected by ensuring through cause–effect analysis that the HOWs are the first-order causes for the WHATs (Ref. [5]). Assume that \( N \) technical measures have been developed, denoted as \( H_1, H_2, \ldots, H_N \). Their measurement units and improving directions should also be determined, which is usually easy to do and important for the company to conduct technical competitive analysis for the HOWs. If necessary, these HOWs could be organized into some manageable categories using, e.g., the Affinity Diagram method (Refs. [2,6,7]).

**Step 6. Determine the relationships between HOWs and WHATs:** This is an important work in HOQ/QFD which is performed carefully and collectively by technicians. The relationship between a HOW and a WHAT is usually determined by analyzing to what extent the HOW could technically relate to and influence the WHAT. All these relationships form a matrix with the WHATs as rows and the HOWs as columns. It is suitable to complete this matrix in a column- or HOW-wise manner since once a HOW is defined we usually begin establishing to what extents it relates to the WHATs (Ref. [2]). Let the relationship value between technical measure \( H_n \) and customer need \( W_m \) be determined as \( r_{mn} \) according to scale (10). Then we can form the following relationship matrix between the HOWs and the WHATs:

\[
R = \begin{bmatrix}
W_1 & W_2 & \cdots & W_M \\
H_1 & r_{11} & \cdots & r_{1N} \\
H_2 & r_{21} & \cdots & r_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
H_N & r_{N1} & \cdots & r_{NN}
\end{bmatrix}_{M \times N}
\]

**Step 7. Determine initial technical ratings of HOWs:** Initial technical ratings of HOWs are decided by two factors, final importance ratings of WHATs and the relationships between the HOWs and the WHATs. These ratings indicate the basic importance of the HOWs developed in relation to the WHATs. They are usually computed using the simple additive weighting (SAW) formula (7). That is, for technical measure \( H_m \), its initial technical rating is computed as the following simple weighted average over its relationships with the WHATs:

\[
t_m = f_1 \times r_{1m} + f_2 \times r_{2m} + \cdots + f_M \times r_{Mm} = \sum_{m=1}^{M} f_m \times r_{mn}, \quad n = 1, 2, \ldots, N.
\]  

(14)

Other methods to obtain comprehensive ratings for a set of choices in relation to a number of performance criteria, such as the technique for order preference by similarity to ideal solution (TOPSIS) (Refs. [21,22]) and the operational competitiveness rating (OCRA) procedure (Refs. [23,24]), can also be used to compute initial technical ratings (Ref. [15]). We will denote, in any case, the HOWs’ initial technical ratings by a vector, \( t = (t_1, t_2, \ldots, t_N) \).

**Step 8: Perform technical competitive analysis:** This step can be done through marketing. Although some technical parameters and know-how of the competitors’ products cannot be easily obtained and some may even be kept confidential, the producing company should make every effort to acquire this information and failing to do so may result in an unfavorable position for the company in the market place. In case of extreme difficulty in obtaining the technical parameters of the competitors’ products on some HOWs, careful technical assessments should be made to give reliable scores (in a suitable scale such as (9)) representing the technical performance of the competitors’ products on the said HOWs.
Let the technical parameter or performance score of company $C_i$’s product on technical measure $H_n$ be determined as $y_{ni}$. Then we can form the technical comparison matrix of the companies’ products on the HOWs:

$$
Y = \begin{bmatrix}
  y_{11} & y_{12} & \cdots & y_{1L} \\
y_{21} & y_{22} & \cdots & y_{2L} \\
\vdots & \vdots & \ddots & \vdots \\
y_{N1} & y_{N2} & \cdots & y_{NL}
\end{bmatrix}_{N \times L}
$$

From this $Y$ information technical competitive priority ratings on the HOWs can be obtained for the producing company using the entropy method as described in the Appendix. We will denote these ratings as a vector, $z = (z_1, z_2, \ldots, z_N)$, where $z_n$ represents the company’s technical competitive priority with respect to $H_n$.

Based on the above matrix $Y$, company $C_1$ could also set performance goals on the HOWs. It should be noted that these goals are different from design specifications. Essentially they represent levels of performance on the HOWs which the company believes is required for its product to be of technical competitiveness in the relevant markets in comparison with its competitors’ similar products. The goals should also be reachable according to the company’s technical resources. Suppose that the company sets a goal performance level $b_n$ for its product on $H_n$, then we have a technical performance goal vector, $b = (b_1, b_2, \ldots, b_N)$. Compared to these goals we can define improvement ratios $v_{n}$’s for the current performance of company’s product on the HOWs:

$$
v_n = \frac{b_n}{y_{n1}} \text{ for } H_n \text{ to be maximized, or when } b_n \geq y_{n1} \text{ for } H_n \text{ to meet target;}$$

$$
v_n = \frac{y_{n1}}{b_n} \text{ for } H_n \text{ to be minimized, or when } b_n < y_{n1} \text{ for } H_n \text{ to meet target,}
$$
or in a uniform manner:

$$
v_n = \max\{y_{n1}, b_n\}/\min\{y_{n1}, b_n\}. \tag{15}
$$

**Step 9. Obtain final technical ratings of the HOWs**: Those HOWs with higher initial technical ratings ($t_n$’s), higher technical competitive priorities ($z_n$’s) and higher improvement ratios ($v_n$’s) indicate working focuses and market opportunities for the producing company. Final technical rating is a useful measure to reflect this point which, with respect to $H_n$, can be computed for the company’s product by integrating all these factors using a formula similar to (5) or (13) for computing the final importance ratings for the WHATs:

$$
s_n = v_n \times t_n \times z_n, \quad n = 1, 2, \ldots, N. \tag{16}
$$

Thus we have a final technical rating vector on the HOWs, $s = (s_1, s_2, \ldots, s_N)$. HOWs with higher final technical ratings, implying greater importance for the company’s product to be successful in the competitive markets, are transferred into the second phase of QFD, parts deployment, which translates important technical measures (new WHATs) into parts characteristics (new HOWs).

The above quantitative descriptions of our proposed HOQ model are summarized in Fig. 2. In the next Section, we will demonstrate step by step the concepts and operations of the model through an easy-to-understand fried Chinese vegetable example. To make our HOQ model fully operable, we will use fuzzy method to handle the vagueness of people’s linguistic assessments and entropy method to derive competitive priority ratings for the company’s product. A brief introduction to these two methods is given in the Appendix.

4. A full illustrative example

Complete QFD examples to fully illustrate the procedure of QFD do not appear frequently in the literature. But they are helpful for practitioners to follow. Here we present a fried Chinese vegetable example to illustrate the concepts and computations in our proposed HOQ model in details. A Chinese restaurant, called restaurant $C_1$, wishes to make an improvement on a fried Chinese vegetable it cooks and sells everyday in response to the competition of other Chinese restaurants in the same district. HOQ technique can help $C_1$ make the appropriate decision resulting in better improvement. The basic idea is (i) to understand what are customer needs for a fried Chinese vegetable (such as “not greasy” and “fresh”) and then to identify the important ones through customer surveys, and (ii) to associate the customer needs with appropriate technical measures or solutions (such as “amount of edible oil used” and “duration of vegetable storage”) and then to find the important ones through technical analyses. In what follows we will build the HOQ model for this example step by step according to the qualitative and quantitative descriptions in Section 3.
Ten customer needs (WHATs) for a fried Chinese vegetable

<table>
<thead>
<tr>
<th>Good taste</th>
<th>Good smell</th>
<th>Good appearance</th>
</tr>
</thead>
<tbody>
<tr>
<td>W_1: appetizing</td>
<td>W_5: hot</td>
<td>W_5: not overcooked</td>
</tr>
<tr>
<td>W_2: not salty</td>
<td>W_6: fresh</td>
<td>W_6: jade color</td>
</tr>
<tr>
<td>W_3: not greasy</td>
<td>W_7: sweet smell</td>
<td>W_10: tasteful arrangement</td>
</tr>
<tr>
<td>W_4: moderately spicy</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Step 1:** At first the restaurant must know who are the customers for its fried Chinese vegetable. This information could be obtained by observing who order the fried Chinese vegetable or through market survey. According to Refs. [7,8], usually 20–30 customers should be selected to reveal their various perceptions about the product. Here, for illustration purpose, five of the restaurant’s customers are selected to help conduct the HOQ analysis (i.e., \(K = 5\)). By one or more focus groups, these five customers identify ten needs (WHATs) for the fried Chinese vegetable (i.e., \(M = 10\)). They are: “appetizing”, “jade color”, “fresh”, “hot” (high temperature), “moderately spicy”, “not greasy”, “not overcooked”, “tasteful arrangement”, “not salty”, and “sweet smell”. Using the Affinity Diagram method or through experience, the customers further group these 10 WHATs into three convenient and meaningful categories according to their inter-relationships: “good taste”, “good smell” and “good appearance”. Under these groupings, the customer needs can be re-ordered and numbered as shown in Table 1.

**Step 2:** The 10 WHATs can hardly be of same importance to the customers. So the TVVvve selected customers are asked to reveal their perceptions on the relative importance of the ten WHATs using the five linguistic terms in scale (8). Table 2(a) shows their assessments where, for example, customer 1 rates the relative importance of \(W_1\) as “high”. Using scale (8), these linguistic assessments of the WHATs’ relative importance can be converted to crisp numbers or symmetrical triangular fuzzy numbers (STFNs) according to practical need. The transformed results are shown in Table 2(b) where, for example, customer 1 considers \(W_1\) as having “high” importance, which can be represented by a crisp number 7 or an STFN [6,8] according to scale (8).

According to these crisp numbers or STFNs representing the relative importance of the WHATs perceived by the customers, we can obtain the relative importance ratings of the WHATs by averaging the customers’ perceptions. In case...
that crisp numbers are used, these relative importance ratings are computed by (11) as
\[ g = (g_1, g_2, \ldots, g_{10}) = (7, 4, 6, 2, 6, 2, 5, 4, 0, 4, 5, 6, 2, 3, 4, 2, 6). \]

For example, the crisp form relative importance rating of \( W_1, g_1 \), is computed by
\[ g_1 = (g_{11} + g_{12} + g_{13} + g_{14} + g_{15})/5 = (7 + 9 + 7 + 5 + 9)/5 = 37/5 = 7.4, \]
where \( g_{mk} \) is customer \( k \)’s relative importance perception on \( W_m \) in crisp form. If STFNs are used, the relative importance ratings are computed by (11) and the arithmetic for STFNs as
\[ \tilde{g} = (\tilde{g}_1, \tilde{g}_2, \ldots, \tilde{g}_{10}) = ([6, 8], [5, 2, 7, 2], [5, 2, 7, 2], [4, 4, 6, 4], [3, 0, 5, 0], [6, 4, 8, 4],
[4, 8, 6, 8], [5, 2, 7, 2], [2, 4, 4, 4], [1, 6, 3, 6]), \]
where, for example, the STFN form relative importance rating of \( W_1, \tilde{g}_1 \), is computed by
\[ \tilde{g}_1 = (\tilde{g}_{11} + \tilde{g}_{12} + \tilde{g}_{13} + \tilde{g}_{14} + \tilde{g}_{15})/5 = ([6, 8] + [8, 10] + [6, 8] + [4, 6] + [8, 10])/5 = [32, 42]/5 = [6, 4, 8, 4], \]
and \( \tilde{g}_{mk} \) is customer \( k \)’s relative importance perception on \( W_m \) in STFN form.

**Step 3:** This step is for restaurant \( C_1 \) to identify competitors and conduct customer competitive analysis. In the district’s fried Chinese vegetable market, restaurant \( C_1 \) has three main competitors, called restaurants \( C_2, C_3 \) and \( C_4 \), each of which makes a similar type of fried Chinese vegetable. In order to understand the fried Chinese vegetable market and its relative position in the market, and to finally find out the priorities for further improvement, restaurant \( C_1 \) asks the five selected customers to rate the relative performance of its own fried Chinese vegetable and the three competitors’ similar products in terms of the ten WHATs using scale (9). The five customers’ assessments are shown in Table 3(a) in crisp numbers, where, for example, customer 1 rates the performance of \( C_2 \)’s product on \( W_1 \) as “neutral” using scale (9), which
Table 3

(a) Customer competitive analysis

<table>
<thead>
<tr>
<th>$W_m$</th>
<th>Customer 1, $X_1 = [x_{mn}]_{10 \times 4}$</th>
<th>Customer 2, $X_2 = [x_{mn2}]_{10 \times 4}$</th>
<th>Customer 3, $X_3 = [x_{mn3}]_{10 \times 4}$</th>
<th>Customer 4, $X_4 = [x_{mn4}]_{10 \times 4}$</th>
<th>Customer 5, $X_5 = [x_{mn5}]_{10 \times 4}$</th>
<th>Customer comparison matrix, $X = [x_{mn}]_{10 \times 4}$</th>
<th>$e_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_1$</td>
<td>5 7 5 3</td>
<td>7 3 7 7</td>
<td>9 5 7 5</td>
<td>5 5 7 5</td>
<td>7 3 7 7</td>
<td>6.6 4.6 5.8 5.4</td>
<td>0.1000</td>
</tr>
<tr>
<td>$W_2$</td>
<td>3 3 5 5</td>
<td>5 7 3 5</td>
<td>3 5 3 7</td>
<td>7 5 5 3</td>
<td>5 5 7 5</td>
<td>4.6 5.0 4.6 5.0</td>
<td>0.1006</td>
</tr>
<tr>
<td>$W_3$</td>
<td>7 5 3 7</td>
<td>9 5 7 5</td>
<td>7 7 5 5</td>
<td>5 3 5 3</td>
<td>7 5 7 5</td>
<td>7.0 5.0 5.0 5.4</td>
<td>0.0999</td>
</tr>
<tr>
<td>$W_4$</td>
<td>3 5 7 5</td>
<td>1 3 5 5</td>
<td>5 5 3 7</td>
<td>7 5 5 3</td>
<td>5 5 5 5</td>
<td>4.2 4.2 5.0 5.0</td>
<td>0.1004</td>
</tr>
<tr>
<td>$W_5$</td>
<td>1 3 1 7</td>
<td>3 1 1 3</td>
<td>1 5 3 1</td>
<td>5 5 3 3</td>
<td>3 5 5 5</td>
<td>2.6 3.4 2.6 2.2</td>
<td>0.0997</td>
</tr>
<tr>
<td>$W_6$</td>
<td>7 5 3 5</td>
<td>9 7 7 7</td>
<td>5 3 5 3</td>
<td>7 7 3 7</td>
<td>7 3 5 5</td>
<td>7.0 5.0 4.6 5.4</td>
<td>0.0997</td>
</tr>
<tr>
<td>$W_7$</td>
<td>9 7 9 5</td>
<td>7 9 7 7</td>
<td>5 3 3 7</td>
<td>7 7 7 3</td>
<td>9 7 7 9</td>
<td>7.4 6.6 6.6 5.8</td>
<td>0.1004</td>
</tr>
<tr>
<td>$W_8$</td>
<td>3 3 7 3</td>
<td>5 3 5 5</td>
<td>3 5 5 5</td>
<td>1 5 7 5</td>
<td>7 5 7 5</td>
<td>3.8 4.2 6.2 4.2</td>
<td>0.0992</td>
</tr>
<tr>
<td>$W_9$</td>
<td>5 3 5 7</td>
<td>7 7 7 5</td>
<td>7 3 5 3</td>
<td>5 5 7 7</td>
<td>3 3 3 5</td>
<td>5.4 4.2 5.4 5.4</td>
<td>0.1003</td>
</tr>
<tr>
<td>$W_{10}$</td>
<td>7 7 3 5</td>
<td>9 7 5 5</td>
<td>7 5 7 3</td>
<td>5 7 5 5</td>
<td>7 9 7 7</td>
<td>7.0 7.0 5.4 5.0</td>
<td>0.0998</td>
</tr>
</tbody>
</table>

(b) Goals and improvement ratios for the 10 WHATs

<table>
<thead>
<tr>
<th>WHATs</th>
<th>Customer comparison matrix, $X = [x_{mn}]_{10 \times 4}$</th>
<th>Goals for WHATs ($a_m$)</th>
<th>Improvement ratios for WHATs ($a_m = a_m/x_{m1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_1$</td>
<td>6.6 4.6 5.8 5.4</td>
<td>7</td>
<td>1.0606</td>
</tr>
<tr>
<td>$W_2$</td>
<td>4.6 5.0 4.6 5.0</td>
<td>5</td>
<td>1.0870</td>
</tr>
<tr>
<td>$W_3$</td>
<td>7.0 5.0 5.0 5.4</td>
<td>8</td>
<td>1.1429</td>
</tr>
<tr>
<td>$W_4$</td>
<td>4.2 4.2 5.0 5.0</td>
<td>5</td>
<td>1.1905</td>
</tr>
<tr>
<td>$W_5$</td>
<td>2.6 3.4 2.6 2.2</td>
<td>4</td>
<td>1.5385</td>
</tr>
<tr>
<td>$W_6$</td>
<td>7.0 5.0 4.6 5.4</td>
<td>8</td>
<td>1.1429</td>
</tr>
<tr>
<td>$W_7$</td>
<td>7.4 6.6 6.6 5.8</td>
<td>9</td>
<td>1.2162</td>
</tr>
<tr>
<td>$W_8$</td>
<td>3.8 4.2 6.2 4.2</td>
<td>5</td>
<td>1.3158</td>
</tr>
<tr>
<td>$W_9$</td>
<td>5.4 4.2 5.4 5.4</td>
<td>6</td>
<td>1.1111</td>
</tr>
<tr>
<td>$W_{10}$</td>
<td>7.0 7.0 5.4 5.0</td>
<td>8</td>
<td>1.1429</td>
</tr>
</tbody>
</table>

 corresponds to a crisp number of 5, i.e., $x_{211} = 5$. We will not consider using STFNs to represent performance assessments since it is too complex to incorporate STFNs into the following entropy computations.

According to the five customers’ assessments of the relative performance of the four restaurants’ similar products in terms of the 10 WHATs, a customer comparison matrix $X = [x_{mn}]_{10 \times 4}$ can be obtained by averaging the customers’ assessments. The elements of this matrix are shown in Table 3(a) where, for example, restaurant $C_1$’s performance rating on $W_2$ is computed by (12) as

$$x_{21} = \frac{(x_{211} + x_{212} + x_{213} + x_{214} + x_{215})/5 = (3 + 5 + 3 + 7 + 5)/5 = 23/5 = 4.6},$$

where $x_{mnk}$ is customer $k$’s assessment of restaurant $C_i$’s performance on $W_m$.

Applying the entropy method as illustrated in the Appendix, we can obtain restaurant $C_i$’s competitive priority ratings on the 10 customer needs based on the above customer comparison matrix $X$. For example, the “distribution” of $W_1$ on the four restaurants’ products is composed of the four restaurants’ performance ratings on $W_1$: (6.6, 4.6, 5.8, 5.4), which is the first row of matrix $X$. Then we can compute the total score of $W_1: x_1 = x_{11} + x_{12} + x_{13} + x_{14} = 6.6 + 4.6 + 5.8 + 5.4 = 22.4$, and obtain the “probability distribution” of $W_1$:

$$p_{11} = x_{11}/x_1 = 6.6/22.4 = 0.2946, \quad p_{12} = x_{12}/x_1 = 4.6/22.4 = 0.2054,$$

$$p_{13} = x_{13}/x_1 = 5.8/22.4 = 0.2589, \quad p_{14} = x_{14}/x_1 = 5.4/22.4 = 0.2411.$$ 

The entropy of $W_1$ is then computed using (A.2) as

$$E(W_1) = -\phi_4 \sum_{i=1}^{4} p_{1i} \ln(p_{1i}) = -\{(0.2946 \ln(0.2946) + 0.2054 \ln(0.2054) + 0.2589 \ln(0.2589) + 0.2411 \ln(0.2411))\}/\ln(4) = 0.9940.$$
We can obtain in the same way the entropy for each of the 10 customer needs as
\[ (E(W_1), E(W_2), \ldots, E(W_{10})) \]
\[ = (0.9940, 0.9994, 0.9925, 0.9973, 0.9909, 0.9905, 0.9973, 0.9859, 0.9961, 0.9919). \]

Finally, according to (A.3) we can obtain restaurant C’s competitive priority ratings on the Wj’s:
\[ e = (e_1, e_2, \ldots, e_{10}) = (0.1000, 0.1006, 0.0999, 0.1004, 0.0997, 0.0997, 0.1004, 0.0992, 0.1003, 0.0998), \]
where, for example,
\[ e_1 = E(W_1) / \sum_{m=1}^{10} E(W_m) \]
\[ = 0.9940 / (0.9940 + 0.9994 + \cdots + 0.9919) = 0.9904/9.9357 = 0.1000. \]

This set of competitive priority ratings are shown in the last column of Table 3(a) from which we know that W2 is of the highest competitive priority for the restaurant, followed by W4, W7 and W6.

Based on the resources available and the relative performance of the four restaurants on the 10 WHATs, restaurant C can set improving goals on each WHAT to better satisfy the customer needs. After various considerations, restaurant C decides the following performance goals on the WHATs using scale (9):
\[ a = (a_1, a_2, \ldots, a_{10}) = (7, 5.8, 5.4, 8, 9, 5, 6, 8). \]

This set of goals is shown in Table 3(b). It is noted that all goal performance levels are higher than C’s current performance levels represented by the first column of customer comparison matrix X. If C’s performance on a WHAT is poorer or much poorer than the performance of most of its competitors, then the goal level is set to be much higher than its current level to be of competitiveness. Otherwise, if C’s performance on a WHAT is better than the performance of most of its competitors, then the goal level is only set to be slightly higher than its current level which is enough for C to keep and enhance its established competitiveness. We do not consider setting goals in STFN form either, since this results in some computational and explanatory difficulties.

According to restaurant C’s current and goal performance levels on the ten WHATs, its improvement ratios with respect to the customer needs can be easily computed according to the formula \( u_m = a_m / x_m \):
\[ u = (u_1, u_2, \ldots, u_{10}) = (1.0606, 1.0870, 1.1429, 1.1905, 1.5385, 1.1429, 1.2162, 1.3158, 1.1111, 1.1429). \]

**Step 4**: According to each WHAT’s relative importance rating, competitive priority rating and improvement ratio, restaurant C could now reach the final importance rating of the WHAT using (13). In case that the relative importance ratings are crisp numbers, the final importance ratings are also given as the following crisp numbers:
\[ f = (f_1, f_2, \ldots, f_{10}) \]
\[ = (0.7852, 0.6779, 0.7078, 0.6453, 0.6137, 0.8431, 0.7081, 0.8095, 0.3787, 0.2966). \]

Here, for example, the final importance rating of W1 in crisp form, \( f_1 \), is computed by (13) as
\[ f_1 = u_1 \times g_1 \times e_1 = 1.0606 \times 7.4 \times 0.1000 = 0.7852. \]

From \( f \) we can finally rank the importance of the ten WHATs in the following order:
\[ W_6 \succ W_5 \succ W_1 \succ W_7 \succ W_3 \succ W_2 \succ W_4 \succ W_5 \succ W_6 \succ W_{10} \]
(17)
where “\( \succ \)” means “more important than”.

If relative importance ratings are STFNs, final importance ratings are also given as STFNs:
\[ \hat{f} = (\hat{f}_1, \hat{f}_2, \ldots, \hat{f}_{10}) = ([0.6791, 0.8913], [0.5685, 0.7872], [0.5937, 0.8220], [0.5258, 0.7647], [0.4603, 0.7671], [0.7292, 0.9570], [0.5860, 0.8302], [0.6789, 0.9400], [0.2673, 0.4901], [0.1825, 0.4107]). \]

Here, for example, the final importance rating of W1 in STFN form, \( \hat{f}_1 \), is computed by (13) and the scalar multiplication rule of STFNs as
\[ \hat{f}_1 = u_1 \times \hat{g}_1 \times e_1 = 1.0606 \times [6.4, 8.4] \times 0.1000 = [0.6791, 0.8913]. \]
Using the fuzzy ranking principle (see Appendix), these fuzzy ratings produce the following partial ranking order for the WHATs’ final importance:

\[ W_6 \succ \{W_8, W_1\} \succ \{W_7, W_3\} \succ W_2 \succ \{W_4, W_5\} \succ W_9 \succ W_{10}, \tag{18} \]

where \( \{W_8, W_1\} \) means that \( W_8 \) and \( W_1 \) are not so easy to be compared, so do \( \{W_7, W_3\} \) and \( \{W_4, W_5\} \). Although the crisp and fuzzy final importance ratings for the ten WHATs may not result in the same ranking order due to possible incomparability between the fuzzy ratings of \( W_8 \) and \( W_1 \), of \( W_7 \) and \( W_3 \), and of \( W_4 \) and \( W_5 \), it is quite clear from (17) and (18) that the two sets of ratings show an identical trend. Both sets of ratings indicate that \( W_6 \) is the most important WHAT, followed by \( W_8 \) and \( W_1 \), and then by \( W_7 \) and \( W_3 \).

These final importance ratings of the WHATs, expressed as both crisp numbers and STFNs, are shown in the second column of Table 4. In order to be comparable, the crisp and fuzzy final importance ratings are both scaled to have maximum ratings of unity, which are also shown in Table 4. More clearly, the scaled ratings are obtained for crisp case by dividing all the ratings by their maximum, 0.8431, and for fuzzy case by dividing all the STFNs by the maximum of their upper limits, 0.9570. From these scaled ratings we can see that, although the crisp and fuzzy ratings result in the same ranking order, crisp ratings are very close to the upper limits of the corresponding fuzzy ratings and quite far away from the lower limits. This shows that fuzzy ratings are more representative of the variations of the WHATs’ importance, which would make the restaurant more flexible to capture and satisfy the customer needs for a fried Chinese vegetable.

**Step 5:** Now it is time to convert customer needs into product design specifications. After careful considerations, restaurant \( C_1 \)’s cooks propose nine technical measures (HOWs) that relate to and can help realize the ten WHATs. These HOWs can be grouped into two categories, “cooking material” and “cooking quality”. Further, the improving direction for each HOW is also determined by experience or technical analysis: two HOWs are to be minimized and the remaining seven HOWs are to meet targets. The HOWs, their improving directions and measurement units are shown in Table 5.

**Step 6:** Then the cooks begin to establish the relationships between the HOWs and the WHATs, or to examine to what extent each HOW is related to and can technically measure each WHAT. This step is usually done simultaneously with Step 5 since in the process of generating HOWs, each HOW’s relationships with the WHATs are always examined once the HOW is considered. The relationships between the HOWs and the WHATs are determined by technical analysis.

### Table 4
Crisp and fuzzy final importance ratings of the 10 WHATs

<table>
<thead>
<tr>
<th>WHATs</th>
<th>Final importance ratings</th>
<th>Scaled final importance ratings</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Crisp ((f_m))</td>
<td>Fuzzy ((f'_m))</td>
</tr>
<tr>
<td>(W_1)</td>
<td>0.7852</td>
<td>[0.6791, 0.8913]</td>
</tr>
<tr>
<td>(W_2)</td>
<td>0.6779</td>
<td>[0.5685, 0.7872]</td>
</tr>
<tr>
<td>(W_3)</td>
<td>0.7078</td>
<td>[0.5937, 0.8220]</td>
</tr>
<tr>
<td>(W_4)</td>
<td>0.6453</td>
<td>[0.5258, 0.7647]</td>
</tr>
<tr>
<td>(W_5)</td>
<td>0.6137</td>
<td>[0.4603, 0.7671]</td>
</tr>
<tr>
<td>(W_6)</td>
<td>0.8431</td>
<td>[0.7292, 0.9570]</td>
</tr>
<tr>
<td>(W_7)</td>
<td>0.7081</td>
<td>[0.5860, 0.8302]</td>
</tr>
<tr>
<td>(W_8)</td>
<td>0.8095</td>
<td>[0.6789, 0.9400]</td>
</tr>
<tr>
<td>(W_9)</td>
<td>0.3787</td>
<td>[0.2673, 0.4901]</td>
</tr>
<tr>
<td>(W_{10})</td>
<td>0.2966</td>
<td>[0.1825, 0.4107]</td>
</tr>
</tbody>
</table>

Using the fuzzy ranking principle (see Appendix), these fuzzy ratings produce the following partial ranking order for the WHATs’ final importance:

\[ W_6 \succ \{W_8, W_1\} \succ \{W_7, W_3\} \succ W_2 \succ \{W_4, W_5\} \succ W_9 \succ W_{10}, \]

### Table 5
Nine technical measures (HOWs) for the 10 customer needs (WHATs)

<table>
<thead>
<tr>
<th>Cooking material</th>
<th>Cooking quality</th>
</tr>
</thead>
<tbody>
<tr>
<td>(H_1): [ (\leftrightarrow)] amount of soy sauce used (ml)</td>
<td>(H_6): [ (\downarrow)] duration of vegetable storage (hour)</td>
</tr>
<tr>
<td>(H_2): [ (\leftrightarrow)] amount of salt used (g)</td>
<td>(H_7): [ (\downarrow)] duration between finished and served (minute)</td>
</tr>
<tr>
<td>(H_3): [ (\leftrightarrow)] amount of pepper used (mg)</td>
<td>(H_4): [ (\leftrightarrow)] duration of cooking (minute)</td>
</tr>
<tr>
<td>(H_5): [ (\leftrightarrow)] amount of vegetable used (g)</td>
<td>(H_5): [ (\leftrightarrow)] duration of high temperature used in cooking (second)</td>
</tr>
</tbody>
</table>

Note: “\(\leftrightarrow\)” represents to meet a target and “\(\downarrow\)” to minimize/decrease.
and empirical judgement, and usually may not be precise. So it is quite appropriate to use STFNs to represent this kind of relationships. For each HOW with respect to each WHAT, the cooks determine the relationship first in linguistic term using scale (10) and then convert this relationship into corresponding crisp number and STFN. The full matrix of relationships. For each HOW with respect to each WHAT, the cooks determine the relationship first in linguistic

Step 7: According to the WHATs’ final importance ratings and the relationship values between the HOWs and the WHATs, the HOWs’ initial technical ratings can be computed usually through the simple additive weighting (SAW) formula (7). When crisp numbers are used, the initial technical ratings are given as

\[ t = (t_1, t_2, \ldots, t_9) = (34.1866, 33.9720, 18.8076, 17.9852, 39.3707, 23.5959, 23.8702, 31.3584, 32.6375). \]

Here, for example, crisp initial technical rating of \( H_1 \), \( t_1 \), is computed as the weighted average over \( H_1 \)’s crisp relationship values with the ten WHATs, \( r_{11}, r_{12}, \ldots, r_{110} \), which correspond to the first column of the crisp relationship matrix \( R \), and the weights are the crisp final importance ratings of the ten WHATs, \( f_1, f_2, \ldots, f_{10} \), i.e.,

\[ t_1 = \sum_{m=1}^{10} f_m \times r_{m1} = 0.7852 \times 9 + 0.6779 \times 7 + \cdots + 0.2966 \times 1 = 34.1866. \]

From these crisp initial technical ratings, the technical measures (HOWs) can be ranked in the following order:

\[ H_5 \succ H_1 \succ H_2 \succ H_6 \succ H_8 \succ H_7 \succ H_6 \succ H_5 \succ H_4. \]  

(19)

If STFNs are used, the initial technical ratings are also given as STFNs:

\[ \bar{t} = (\bar{t}_1, \bar{t}_2, \ldots, \bar{t}_9) = ([23.2850, 47.4773], [23.0639, 47.2691], [9.9597, 30.0446], [9.1375, 29.2221], [27.0869, 54.0437], [13.7766, 35.8043], [13.8342, 36.2954], [20.3964, 44.7095], [21.3973, 46.2669]). \]

Here, for example, the initial technical rating of \( H_1 \) in STFN form, \( \bar{t}_1 \), is computed as the weighted average over \( H_1 \)’s STFN form relationship values with the ten WHATs, \( \tilde{r}_{11}, \tilde{r}_{12}, \ldots, \tilde{r}_{110} \), which correspond to the first column of the STFN form relationship matrix \( \tilde{R} \), and the weights are the final importance ratings of the ten WHATs in STFN form, \( \tilde{f}_1, \tilde{f}_2, \ldots, \tilde{f}_{10} \), i.e.,

\[ \bar{t}_1 = \sum_{m=1}^{10} \tilde{f}_m \times \tilde{r}_{m1} = [0.6791, 0.8913] \times [8, 10] + [0.5685, 0.7872] \times [6, 8] + \cdots + [0.1825, 0.4107] \times [0, 2] \]

\[ = [23.2850, 47.4773]. \]

According to the principle in the Appendix, these fuzzy ratings have the following ranking order for the HOWs’ initial importance:

\[ H_5 \succ H_1 \succ H_2 \succ H_6 \succ H_8 \succ H_7 \succ H_6 \succ H_5 \succ H_4. \]  

(20)
It is noticed from (19) and (20) that the crisp and fuzzy ratings exhibit the same ranking order. Both sets of ratings indicate that \( H_5 \) is of the highest initial importance, followed by \( H_1 \), \( H_2 \) and \( H_9 \).

The crisp and fuzzy initial technical ratings of the nine HOWs are shown in Table 7. Also shown there are the scaled crisp and fuzzy ratings that are easier to be compared. Same as in the case for the WHATs’ final importance ratings (Table 4), crisp initial technical ratings tend to be close to the upper bounds and far away from the lower bounds of the corresponding fuzzy ratings, indicating more flexibility and higher reliability represented by the fuzzy ratings.

Step 8: Now turn to technical competitive analysis which is to find and establish competitive advantages or to further enhance the existing advantages for restaurant \( C_1 \), through comparing all the restaurants’ similar fried Chinese vegetables in terms of their technical performance on the nine identified HOWs. Although it is always not easy to acquire the technical performance levels of competitors’ products on the HOWs, restaurant \( C_1 \) must try all the means to obtain this valuable information in order to know its technical strengths and weaknesses and hence to improve or enhance its competitiveness.

Through a lot of efforts restaurant \( C_1 \) obtains all the technical parameters of its own and its competitors’ fried Chinese vegetables in terms of the nine HOWs. This information forms a technical comparison matrix \( Y = [y_{nl}]_{9 \times 4} \) as shown in Table 8 where, for example, amount of soy sauce the four restaurants use to make fried Chinese vegetables (\( H_1 \)) are 11, 13, 8 and 9 ml, respectively, which form the first column of the technical comparison matrix \( Y \).

Applying entropy method to \( Y \) in the same manner as in customer competitive analysis (Step 3), technical competitive priority ratings can be obtained for restaurant \( C_1 \)’s fried Chinese vegetable on the nine HOWs:

\[
\mathbf{z} = (z_1, z_2, \ldots, z_9) = (0.1115, 0.1116, 0.1104, 0.1124, 0.1119, 0.1101, 0.1104, 0.1125, 0.1093).
\]

These ratings are shown in Table 8 from which we know that \( H_6 \) and \( H_4 \) are of the highest competitive priorities.

According to the technical performance of its own and the other three restaurants’ fried Chinese vegetables in terms of the nine HOWs, restaurant \( C_1 \) could set technical performance goal on each of the HOWs for its fried Chinese vegetable
to better fulfill the customer needs. These goals should be determined both competitively and realistically. Suppose that restaurant C1’s relevant personnel agrees with the following performance goals on the HOWs for further improvement:

\[ b = (b_1, b_2, \ldots, b_9) = (9, 8, 25, 300, 9, 6, 2, 3.5, 15). \]

From these goal \((b_i)\) and current \((y_{ni})\) technical performance levels, improvement ratios for restaurant C1’s fried Chinese vegetable to be competitive in terms of the HOWs can be easily computed using (15) as

\[ r = (r_1, r_2, \ldots, r_9) = (1.2222, 1.3333, 1.2500, 1.2000, 1.2857, 1.3333, 1.2500, 1.1667, 1.3333). \]

These goals and improvement ratios are shown in the last two columns of Table 8.

**Step 9:** This is the last step of our proposed HOQ model. Integrating the initial technical ratings, technical competitive priority ratings and improvement ratios of the HOWs, final technical ratings can be computed by (16). If initial technical ratings are crisp numbers, the final technical ratings are also crisp numbers and given as

\[ s = (s_1, s_2, \ldots, s_9) = (4.6595, 5.0541, 2.5948, 2.4250, 5.6622, 3.4637, 3.2933, 4.1163, 4.7578). \]

Here, for example, the final technical rating of \(H_1\) in crisp form, \(s_1\), is computed by (16) as

\[ s_1 = v_1 \times t_1 \times z_1 = (y_{11}/b_1) \times t_1 \times z_1 = (11/9) \times 34.1866 \times 0.1115 = 4.6595. \]

From \(s\) we can rank the final technical importance of the nine HOWs in the following order:

\[ H_5 \succ H_2 \succ H_9 \succ H_1 \succ H_8 \succ H_6 \succ H_7 \succ H_3 \succ H_4. \]  

(21)

This final technical importance order differs from the initial technical importance order (19) in two aspects: (i) \(H_1\) is of higher initial technical importance but lower final technical importance than \(H_2\) and \(H_8\), and (ii) \(H_8\) is of lower initial technical importance but high final technical importance than \(H_7\). Since technical competitive priority ratings \((z_{ni}\)'s) do not vary too much, these two differences are mainly caused by the setting of performance goals \((b_{ni}\)'s) or improvement ratios \((v_{ni}\)'s): (i) \(H_1\)'s improvement ratio (1.2222) is lower than \(H_2\)'s (1.3333) and \(H_8\)'s (1.3333), and (ii) \(H_8\)'s improvement ratio (1.3333) is higher than \(H_7\)'s (1.2500).

If initial technical ratings are STFNs, then the final technical ratings are also given as STFNs:

\[ \tilde{s} = (\tilde{s}_1, \tilde{s}_2, \ldots, \tilde{s}_9) = \{(3.1736, 6.4709), [3.4313, 7.0324], [1.3741, 4.1451], [2.3227, 3.9402], [3.8956, 7.7724], [2.0223, 5.2558], [1.9083, 5.0075], [2.6774, 5.8689], [3.1192, 6.7447]\}. \]

Here, for example, the final technical rating of \(H_1\) in STFN form, \(\tilde{s}_1\), is computed by (16) and the arithmetic of STFNs as

\[ \tilde{s}_1 = v_1 \times \tilde{t}_1 \times z_1 = (11/9) \times [23.2850, 47.4773] \times 0.1115 = [3.1736, 6.4709]. \]

These fuzzy ratings produce the following ranking order for the HOWs’ final importance:

\[ H_5 \succ H_2 \succ (H_9, H_1) \succ H_8 \succ H_6 \succ H_7 \succ H_3 \succ H_4. \]  

(22)

It is noticed from (21) and (22) that the crisp and fuzzy ratings show an almost identical ranking order for the HOWs’ final technical importance. Both sets of ratings indicate that \(H_5\) is the most important HOW, followed by \(H_2\) and then by \(H_9\) and \(H_1\), and that \(H_8\) is the least important HOW, preceded by \(H_1\) and \(H_2\).

These crisp and fuzzy final technical ratings of the HOWs are shown in Table 9. In order to be comparable, they are both scaled to have maximum rating or upper limit of unity, which are also shown in Table 9. From these scaled ratings we can see again that, although the crisp and fuzzy ratings exhibit an identical trend, crisp ratings always tend to be close to the upper limits of the corresponding fuzzy ratings. This shows that fuzzy ratings are more representative of the possible variations of the HOWs’ technical importance, which would make the technical improvement more flexible and the design process more feasible.

The above nine steps complete the HOQ process for improving the fried Chinese vegetable. The corresponding tables of results, after appropriate arrangement, can form an HOQ like Fig. 2 which links customer needs to technical considerations and exhibits all the relevant elements and their relationships. As a result of this HOQ model, it is concluded that \(H_4\) could be deleted from further consideration (in QFD’s second phase, parts deployment) to save technical efforts without decreasing customer satisfaction. If resource or budget considerations require to further cut down the number of HOWs, \(H_3, H_7\) and \(H_6\) form a good deleting order that will not significantly influence the fulfillment of the customer needs.
Table 9
Crisp and fuzzy final technical ratings of the nine HOWs

<table>
<thead>
<tr>
<th>HOWs</th>
<th>Final technical ratings</th>
<th>Scaled final technical ratings</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Crisp (sₙ)</td>
<td>Fuzzy (₀sₙ)</td>
</tr>
<tr>
<td>H₅</td>
<td>4.6595 [3.1736, 6.4709]</td>
<td>0.8229 [0.4083, 0.8326]</td>
</tr>
<tr>
<td>H₂</td>
<td>5.0541 [3.4313, 7.0324]</td>
<td>0.8926 [0.4415, 0.9048]</td>
</tr>
<tr>
<td>H₃</td>
<td>2.5948 [1.3741, 4.1451]</td>
<td>0.4583 [0.1768, 0.5333]</td>
</tr>
<tr>
<td>H₄</td>
<td>2.4250 [1.2321, 3.9402]</td>
<td>0.4283 [0.1585, 0.5069]</td>
</tr>
<tr>
<td>H₅</td>
<td>5.6622 [3.8956, 7.7724]</td>
<td>1.0000 [0.5012, 1.0000]</td>
</tr>
<tr>
<td>H₆</td>
<td>3.4637 [2.0223, 5.2558]</td>
<td>0.6117 [0.2602, 0.6762]</td>
</tr>
<tr>
<td>H₇</td>
<td>3.2933 [1.9086, 5.0075]</td>
<td>0.5816 [0.2456, 0.6443]</td>
</tr>
<tr>
<td>H₈</td>
<td>4.1163 [2.6774, 5.8689]</td>
<td>0.7270 [0.3445, 0.7551]</td>
</tr>
<tr>
<td>H₉</td>
<td>4.7578 [3.1192, 6.7447]</td>
<td>0.8403 [0.4013, 0.8678]</td>
</tr>
</tbody>
</table>

5. About the probability factors and correlation matrices

As mentioned in Section 2.2, our proposed 9-step HOQ model has two major exclusions to make the model simple and unified. One is the probabilities to achieve the goals set for the HOWs, and one is the two correlation matrices. Certainly these two omitted parts involve some degree of difficulties to be dealt with. But for the completeness of this study, here we provide some considerations about them and suggest some ways to handle them in a potentially more comprehensive HOQ model.

5.1. Probabilities to achieve goals for HOWs

A seemingly natural approach is to directly incorporate these probabilities into formula (16) for calculating the HOWs’ final technical ratings:

$$
\text{final technical rating of a HOW (sₙ)} = \text{improvement ratio (vₙ)} \times \text{initial technical rating (tₙ)} \times \text{technical competitive priority (zₙ)} \times \text{probability to achieve the goal for the HOW}.
$$

However, there is an apparent problem with this approach, i.e., the possibly serious overlapping of information contained in improvement ratios, technical competitive assessments and probability factors. This is because the determination of the probability factors, no matter easy or not, depends largely on improvement ratios and technical competitive assessments. So the direct inclusion of these probabilities into the calculation of the HOWs’ final technical ratings will overestimate or underestimate some underlying factors, resulting in the overestimation or underestimation of some HOWs’ final importance. As a result, we suggest that practitioners employ the simpler formula (16) in our proposed 9-step HOQ model rather than formula (23) to compute the HOWs’ final technical ratings.

5.2. Correlations among WHATs and among HOWs

Correlations among the HOWs are mentioned and included in many QFD studies, but are seldom incorporated into the calculation of the HOWs’ final ratings. Very few studies include or even mention the correlations among the WHATs, not to say to incorporate such correlations into the calculation of the WHATs’ final ratings. One reason for these is the difficulty in obtaining the relevant data since, although they can be measured using appropriate scales such as (1), (6) or (10), correlations among the HOWs and among the WHATs require the technicians and the customers to make a lot of pair-wise comparisons about the degrees to which the HOWs and the WHATs are inter-related. This would be tedious and difficult from the customer’s perspective, although it is possible from the technician’s side.

Another equal important reason is that, although these correlations are useful in analyzing the WHATs and especially the HOWs, there seem few, if any, good methods to incorporate the correlations into the calculation of the WHATs’ and the HOWs’ final ratings. Most of times people simply use formulas (5) and (7) to obtain these final ratings, ignoring the correlations among the HOWs and among the WHATs even if they are available. This seems common not only in QFD, but also in other similar situations. For example, stock index for a given market is simply the weighted average of the
Among the very few approaches found in the QFD literature to dealing with the correlations, a noticeable one is proposed by Khoo and Ho [12]. For a target WHAT (or HOW), the approach determines its importance rating as linear combination of its correlations with other WHATs weighted by the WHATs’ initial importance ratings:

\[
\text{importance rating of a WHAT} = \sum (\text{initial importance rating of other WHAT} \times \text{value of correlation between the target WHAT and other WHAT}),
\]

where the summation is over all WHATs. This method makes some intuitive sense, although it apparently tends to overestimate/underestimate the importance ratings of those WHATs (or HOWs) with positive/negative correlations with other WHATs. Therefore, if we have the required correlations and if we think this approach is appropriate, we may consider the so obtained importance ratings for WHATs (or HOWs) by formula (24) as the “new” initial importance ratings and then employ our formula (13) (or (16) for HOWs) to get the final importance ratings.

6. Conclusions

In the literature, different authors build different house of quality (HOQ) models that contain different elements and employ different scales to measure the relevant concepts, which may puzzle the practitioners as which HOQ models should be used. This paper presents a systematic and operational approach to HOQ to help resolve this problem. We first give a comprehensive description of the relevant elements in HOQ. Then we propose a 9-step HOQ model to unify the HOQ process and a few 9-point scales to unify the measurements in HOQ to avoid arbitrariness and incomparability. We especially address the various “voices” in the HOQ process and suggest the use of symmetrical triangular fuzzy numbers (STFNs) to reflect the vagueness in people’s linguistic assessments. Furthermore, since the commonly used sales point concept is quite subjective, we employ the quantitative entropy method to conduct competitive analysis and derive competitive priority ratings. All information required, computations involved and feasible methods are clearly indicated to give an applicable framework for practitioners to perform HOQ analysis without confusions and difficulties.

To fully illustrate our proposed HOQ model, we present a fried Chinese vegetable example that involves 10 customer needs (WHATs), nine technical measures (HOWs) and five restaurants. Although many studies on QFD have been done and a lot of QFD applications have been reported (see Ref. [25] for a review), there are according to our knowledge few complete QFD cases or examples published. This causes problems for practitioners to apply the QFD technique to their product or service improvement processes. QFD is a complex and time-consuming process that involves many concepts to understand, much information to collect and many computations to perform. A complete and operational description of the QFD process will facilitate its wide applications, and a full example illustrating all the concepts, information collecting, computation and implementation steps will undoubtedly be helpful. Our paper, especially the example presented, is towards such a purpose.

Our proposed 9-step HOQ model, although quite comprehensive and unified in nature, admittedly makes two major exclusions: one is the probability factors for achieving the goals for HOWs, another is the two correlation matrices of the WHATs and the HOWs. Since these two parts involve certain degrees of difficulties, they are omitted to make our model simpler and more operable. Of course, this does not mean they are unimportant for the QFD process. We discuss in some detail the problems associated with the two omitted parts, and also suggest some possible approaches to directly incorporate them into the relevant calculations within a potentially enlarged HOQ model. Further efforts in this aspect through sharing of research and application experiences are highly welcome to make the QFD process more complete.

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Appendix. Fuzzy and entropy methods

A.1. Fuzzy method for vagueness representation

Fuzzy set theory was developed for solving problems in which descriptions of objects are subjective, vague and imprecise, i.e., no boundaries for the objects can be well defined. Let \( X = \{x\} \) be a traditional set of objects, called the universe. A fuzzy set \( \tilde{F} \) in \( X \) is characterized by a membership function \( \mu_{F}(x) \) that associates each object in \( X \) with a membership value in the interval \([0, 1]\), indicating the degree of the object belonging to \( \tilde{F} \).

A fuzzy number is a special fuzzy set when the universe \( \tilde{F} \) is the real line \( \{x\} = (-\infty, +\infty) \). A symmetrical triangular fuzzy number (STFN), denoted as \( \tilde{F} = [a, c] \), is a special fuzzy number with the following symmetrical triangular type of membership function:

\[
\mu_{F}(x) = 1 - \frac{|x - (c + a)/2|}{(c - a)/2}, \quad a \leq x \leq c.
\]

STFN is widely used in practice to represent a fuzzy set or concept \( \tilde{A} \). For example, if customer need \( W_{1} = \) “appetizing” is rated as having “very high” importance by a customer, then traditionally we may assign \( W_{1} \) a number 9 using scale (8). To capture the vagueness of the customer’s subjective assessment, we can according to the same scale (8) assign \( W_{1} \) an STFN \([8,10]\) which means “approximately 9” and is represented by the following membership function:

\[
\mu_{[8,10]}(x) = 1 - \frac{|x - 9|}{1}, \quad 8 \leq x \leq 10.
\]

This means that, for example, the membership value or “possibility” that \( W_{1} = \) “appetizing” is assigned a number 9 is \( \mu_{[8,10]}(9) = 1 \), the “possibility” that \( W_{1} \) is assigned a number 8.5 or 9.5 is \( \mu_{[8,10]}(8.5) = 0.5 \) or \( \mu_{[8,10]}(9.5) = 0.5 \). So assigning \( W_{1} \) a number 8.5 or 9.5 is acceptable or “possible” to the degree of 50%.

The basic arithmetic rules for STFNs are as follows:

\[
\begin{align*}
\text{Addition:} & \quad [a_{1}, c_{1}] + [a_{2}, c_{2}] = [a_{1} + a_{2}, c_{1} + c_{2}], \\
\text{Subtraction:} & \quad [a_{1}, c_{1}] - [a_{2}, c_{2}] = [a_{1} - c_{2}, c_{1} - a_{2}], \\
\text{Scalar multiplication:} & \quad k \times [a_{1}, c_{1}] = [ka_{1}, kc_{1}], \quad k > 0, \\
\text{Multiplication:} & \quad [a_{1}, c_{1}] \times [a_{2}, c_{2}] = [a_{1}a_{2}, c_{1}c_{2}], \quad a_{1} \geq 0, \quad a_{2} \geq 0, \\
\text{Division:} & \quad [a_{1}, c_{1}] \div [a_{2}, c_{2}] = [a_{1}/a_{2}, c_{1}/c_{2}], \quad a_{1} \geq 0, \quad a_{2} > 0.
\end{align*}
\]

For any two STFNs, \( \tilde{F}_{1} = [a_{1}, c_{1}] \) and \( \tilde{F}_{2} = [a_{2}, c_{2}] \), if one interval is not strictly contained by another then their ranking order can be easily and intuitively determined. That is

- If \( c_{2} > c_{1} \) and \( a_{2} \geq a_{1} \), or \( c_{2} \geq c_{1} \) and \( a_{2} > a_{1} \), then \( \tilde{F}_{2} \succ \tilde{F}_{1} \), where \( \succ \) means “is more preferred (important, superior, etc.) than”.
- If \( c_{2} = c_{1} \) and \( a_{2} = a_{1} \), then \( \tilde{F}_{2} \equiv \tilde{F}_{1} \).

But if one interval is strictly contained by another, i.e., if \( c_{2} < c_{1} \) and \( a_{2} > a_{1} \), or \( c_{2} > c_{1} \) and \( a_{2} < a_{1} \), then the ranking problem becomes complex and many possibilities may occur.

For more details about fuzzy set theory, STFNs and fuzzy ranking methods, see Refs. [13,26–29].

A.2. Entropy method for competitive priority ratings

In our HOQ model, Step 3 is to obtain and analyze the following customer comparison matrix:

\[
X = \begin{bmatrix}
W_{1} & \cdots & W_{L} \\
\begin{bmatrix}
x_{11} & x_{12} & \cdots & x_{1L} \\
x_{21} & x_{22} & \cdots & x_{2L} \\
\vdots & \vdots & \ddots & \vdots \\
x_{M1} & x_{M2} & \cdots & x_{ML}
\end{bmatrix}
\end{bmatrix}_{M \times L},
\]

where \( x_{ml} \) is the performance of company \( l \)'s product on customer need \( W_{m} \), perceived by the customers. Based on this \( X \) information, the producing company \( C_{1} \) may set priorities for its product on the \( M \) customer needs in order to achieve a relative competitive advantage over other companies. If company \( C_{1} \) performs much better than many other companies.
in terms of a customer need $W_m$, then further improvement may not be urgently needed and thus a lower priority could be assigned to $W_m$. At the other extreme, if $C_1$ performs much worse than many other companies on $W_m$, then it may be difficult for $C_1$ to build a competitive advantage within a short period of time. In both cases, $W_m$ could be assigned a lower priority rating. However, if most companies perform quite similarly on $W_m$, not too much improvement effort from $C_1$ may result in a better performance of its product and give $C_1$ a unique competitive advantage. Thus a higher priority could be assigned to $W_m$. In particular, if all companies’ performances on $W_m$ are the same, it implies a great market opportunity since any improvement would create a significant competitive advantage. So the highest priority could be assigned to $W_m$. This basis of assigning priorities is interestingly related to the entropy concept in information theory.

Entropy is a measure for the amount of information (or uncertainty, variations) represented by a discrete probability distribution, $p_1, p_2, \ldots, p_L$:

$$E(p_1, p_2, \ldots, p_L) = -\phi_L \sum_{l=1}^{L} p_l \ln(p_l), \quad (A.1)$$

where $\phi_L = 1/\ln(L)$ is a normalization constant to guarantee $0 \leq E(p_1, p_2, \ldots, p_L) \leq 1$. Larger entropy or $E(p_1, p_2, \ldots, p_L)$ value implies smaller variations among the $p_l$’s and hence less information contained in the distribution. For the $m$th row of the customer comparison matrix $X$ corresponding to the customer need $W_m, x_{m1}, x_{m2}, \ldots, x_{mL}$, let $x_m = \sum_{l=1}^{L} x_{ml}$ be the total score with respect to $W_m$. Then according to (A.1), the normalized ratings $p_{ml} = x_{ml}/x_m$ for $l = 1, 2, \ldots, L$ can be viewed as the “probability distribution” of $W_m$ on the $L$ companies with entropy as

$$E(W_m) = -\phi_L \sum_{l=1}^{L} p_{ml} \ln(p_{ml}) = -\phi_L \sum_{l=1}^{L} (x_{ml}/x_m) \ln(x_{ml}/x_m). \quad (A.2)$$

It is clear that the larger the $E(W_m)$ value, the less information contained in $W_m$ or smaller variations among the $p_{ml}$’s (or $x_{ml}$’s). If all companies’ performance ratings on $W_m, x_{m1}, x_{m2}, \ldots, x_{mL}$, are the same, $W_m$ has zero variations and $E(W_m)$ achieves its maximum of 1. So $E(W_m)$ can be used to reflect the relative competitive advantage in terms of the customer need $W_m$. All these $E(W_m)$ values, after normalization:

$$e_m = E(W_m) / \sum_{m=1}^{M} E(W_m), \quad m = 1, 2, \ldots, M \quad (A.3)$$

can be considered as the customer competitive priority ratings for company $C_1$ on the $M$ customer needs, with a larger $e_m$ indicating higher competitive priority for the corresponding $W_m$.

For more on entropy and its applications, see Refs. [13,22,30–32].

References