Abstract

An integrated formulation and solution approach to Quality Function Deployment (QFD) is presented. Various models are developed by defining the major model components (namely, system parameters, objectives, and constraints) in a crisp or fuzzy way using multiattribute value theory combined with fuzzy regression and fuzzy optimization theory. The proposed approach would allow a design team to reconcile tradeoffs among the various performance characteristics representing customer satisfaction as well as the inherent fuzziness in the system. In addition, the modeling approach presented makes it possible to assess separately the effects of possibility and flexibility inherent or permitted in the design process on the overall design. Knowledge of the impact of the possibility and flexibility on customer satisfaction can also serve as a guideline for acquiring additional information to reduce fuzziness in the system parameters as well as determine how much flexibility is warranted or possible to improve a design. The proposed modeling approach would be applicable to a wide spectrum of design problems where multiple design criteria and functional design relationships are interacting and/or conflicting in an uncertain, qualitative, and fuzzy way. © 2000 Elsevier Science B.V. All rights reserved.

Keywords: Fuzzy sets; Multicriteria analysis; Quality function deployment

1. Introduction

Today, many companies are facing rapid changes stimulated by technological innovations and changing customer demands. These companies realize that the effort to develop new products faster that customers want and continue to purchase is crucial for their survival. New product development is a complex process involving multiple functional groups, each with a different perspective. Quality Function Deployment (QFD) is a concept and mechanism for translating the ‘voice of the customer’ through the various stages of product planning, engineering, and manufacturing into a final product.
We propose a fuzzy theoretic modeling approach to QFD by developing and illustrating fuzzy multiobjective models to aid a design team in choosing target levels for engineering characteristics in various environments. These models would allow the team to consider tradeoffs among various performance characteristics, as well as to consider the inherent fuzziness in the relationships linking performance characteristics and engineering characteristics and the engineering characteristics to each other. (“Performance characteristics” refer to the desiderata or product attributes that customers want. The term “customer attributes” has been used for performance characteristics in the QFD literature.)

1.1. Background

The overall objective of QFD is to reduce the length of the product development cycle, while simultaneously improving product quality and delivering the product at a lower cost. Some of the acknowledged advantages of QFD include the preservation of knowledge, fewer startup problems, shorter lead times, warranty claim reduction, development of cross-functional teamwork, easier documentation, and, above all, customer satisfaction (Sullivan, 1986; Eureka, 1987; Bossert, 1991).

QFD was originally developed and implemented in Japan at the Kobe Shipyards of Mitsubishi Heavy Industries in 1972. During the 1970s, Toyota and its suppliers developed QFD further in order to address design problems associated with automobile manufacturing. Hauser and Clausing (1988) observed that Toyota was able to reduce startup and pre-production costs by 60% from 1977 to 1984 through the use of QFD.

During the 1980s many US based companies began employing QFD. It is believed that there are now over 100 major companies using QFD in the US (Griffin and Hauser, 1993), including Motorola, Digital Equipment Corporation, Hewlett Packard, Xerox, AT&T, NASA, Eastman Kodak, Goodyear, Proctor and Gamble, Ford, General Motors (Bossert, 1992; Griffin, 1992; Shipley, 1992), and United States housing industry (Armacost et al., 1994). Many successes across a broad range of industries have been reported at the USA QFD symposia, which have been held annually since 1989 (ASI, 1989–1996).

1.2. Concept

The basic concept of QFD is to translate the desires of the customer into product design or engineering characteristics, and subsequently into parts characteristics, process plans, and production requirements associated with its manufacture. Ideally, each translation uses a chart, called “house of quality” (Fig. 1). A house of quality typically contains information on “what to do” (performance characteristics), “how to do it” (engineering characteristics), the integration of this information (relationships between performance characteristics and engineering characteristics and among the engineering characteristics) and benchmarking data.

It is critically important to capture the customers’ perspective in the corporate language. The customer information comes from a variety of sources, including surveys, focus groups, interviews, listening to salespeople, trade shows and journals, existing data on warranty and customer complaints (Bossert, 1991). Griffin and Hauser (1993) address specific issues on identifying customer needs (how many customers, how many analysts, groups vs. depth interviews), structuring and sorting customer needs, and measuring or estimating relative importance. In practice, over 50% of the QFD effort is spent in capturing the voice of the customer-performance characteristics, relative importance of performance characteristics (Bossert, 1992) etc.

1.3. Setting target engineering characteristic levels

Based upon the information contained in a house of quality, “target levels” for the engineering characteristics of the new (or revised) product are determined. The process of setting the target levels in practice currently is accomplished in a subjective, ad hoc manner, for example, through a team consensus. Given that a house of quality may
contain many performance characteristics and engineering characteristics, it is difficult and lengthy to obtain a feasible competitive design using such a process. Namely, many tradeoffs may have to be made among the performance characteristics, as well as among many implicit or explicit...
relationships interrelating the engineering characteristic levels and the engineering characteristic levels with the performance characteristics. Moreover, such relationships are typically vague and imprecise in practice. The vagueness or impreciseness (that is, fuzziness) arises mainly from the fact that the performance characteristics which tend to be subjective, qualitative, and nontechnical need to be translated into the engineering characteristics which should be expressed in more quantitative and technical terms (Clausing, 1994). Further, data available for product design is often limited, inaccurate, or vague at best (particularly when developing an entirely new product).

Notwithstanding the rapid growth of the QFD literature (see, for example, Khoo and Ho (1996) and Lai et al. (1998) for recent developments in fuzzy modeling and group decision support system for QFD), development of systematic procedures for setting the target engineering characteristic levels has scarcely been addressed. The only prescriptive modeling approaches to QFD to date are found in Wasserman (1993) and Thurston and Locascio (1993). Wasserman (1993) formulated the QFD planning process as a linear programming model to select the mix of design features, which would result in the highest level of customer satisfaction. The model focuses on prioritizing the allocation of resources among design features, rather than determining the target engineering characteristic levels. Thurston and Locascio (1993) interpreted the house of quality as a general formulation of a multiattribute design optimization problem. It was implicitly assumed that the functional relationships between performance characteristics and engineering characteristics always can be identified using engineering knowledge. It would be difficult to justify this assumption in a general situation. Moreover, the interrelationships among the engineering characteristics were not properly considered in the model.

In the next section, the general QFD problem is defined formally, and fuzzy multiobjective models are formulated and illustrated. The results obtained using the models on a small design problem are given in Section 3. Section 4 presents conclusions and some future research directions.

2. Model formulation

2.1. Problem definition

The process of determining target values for the engineering characteristics in QFD can be formulated as an optimization problem. Let $y_i = \text{customer perception of the degree of achievement of performance characteristic } i, i = 1, \ldots, m,\]$ $x_j = \text{target value of engineering characteristic } j, j = 1, \ldots, n,$ $f_i = \text{functional relationship between performance characteristic } i \text{ and engineering characteristics, } i = 1, \ldots, m, \text{ i.e., } y_i = f_i(x_1, \ldots, x_n),$ $g_j = \text{functional relationship between engineering characteristic } j \text{ and other engineering characteristics, } j = 1, \ldots, n, \text{ i.e., } x_j = g_j(x_1, \ldots, x_{j-1}, x_{j+1}, \ldots, x_n).$ A multiobjective optimization model can be formulated as follows:

Find engineering characteristic target values $x_1, x_2, \ldots, x_n$ which

Maximize Overall Customer Satisfaction

for $(y_1, \ldots, y_m)$

subject to

$y_i = f_i(X), \quad i = 1, \ldots, m,$

$x_j = g_j(X'), \quad j = 1, \ldots, n,$

where

$X = (x_1, \ldots, x_n)^T,$ and

$X' = (x_1, \ldots, x_{j-1}, x_{j+1}, \ldots, x_n)^T.$

Additional constraints may be added to the above formulation as appropriate.

2.2. Parameter estimation of functional relationships

A house of quality provides information on the basic relationships between performance characteristics and engineering characteristics and among
engineering characteristics, along with the benchmarking data set. We use such information to estimate the parameters of the functional relationships $f_i$ and $g_j$ in Eqs. (2.2) and (2.3). It is necessary to incorporate both qualitative and quantitative information about interactions among performance characteristics and engineering characteristics into the relationship assessment. In view of this, fuzzy regression (Tanaka et al., 1982) is employed for the estimation purpose in this work.

Fuzzy regression aims to model vague and imprecise phenomena using fuzzy parameters. It has been noted that fuzzy regression may be more effective than statistical regression when the latter’s assumptions are either violated or cannot be properly employed, as for example, when human judgments are involved (Tanaka et al., 1982), ambiguous processes must be explained or its outputs must be predicted (Gharpuray et al., 1986; Heshmaty and Kandel, 1985).

The inherent fuzziness in QFD modeling makes fuzzy regression more appealing than classical statistical tools. The qualitative scales often used to measure the performance characteristic levels, the vagueness or imprecision of the association between the performance characteristics and engineering characteristics are major sources of fuzziness. Moreover, the descriptive validity (i.e., developing a relationship among variables) of fuzzy regression improves over statistical regression as the size of the data set diminishes (Kim et al., 1996), which is often the case in QFD applications. Thus, fuzzy regression may be more appropriate and can yield more reliable system parameter estimates for QFD modeling in the typical industrial environment.

Consider a fuzzy linear function

$$Y = \mathbf{a}^T \mathbf{x} = \sum_{j=1}^{n} a_j x_j = \mathbf{a}^T \mathbf{x},$$

where $Y$ is the dependent variable and $X$ is a vector of the independent variables. $\mathbf{a} = (a_1, a_2, \ldots, a_n)$ are fuzzy parameters, and can be denoted in vector form as $\mathbf{a} = \{ (a_{m1}, a_{m2}, \ldots, a_{mn}), (a_{s1}, a_{s2}, \ldots, a_{sn}) \}$. Here, $a_{mj}$ is the center value of $a_j$, and $a_{sj}$ is the width (or spread) of $a_j$ around $a_{mj}$. A linear symmetric membership function is employed for fuzzy parameters in this discussion for simplicity. The center value describes the most possible value of $a_j$, while the spread represents the precision of $a_j$. The spread essentially defines the support of $a_j$, a set of all the values that have a nonzero membership grade in $a_j$.

The problem in the fuzzy regression model is to determine fuzzy parameter estimates

$$\mathbf{a} = (a_1, a_2, \ldots, a_n) = \{ (a_{m1}, a_{m2}, \ldots, a_{mn}), (a_{s1}, a_{s2}, \ldots, a_{sn}) \}$$

such that the membership value of $y_k$ (the $k$th observed value of the dependent variable) to its fuzzy estimate $\hat{y}_k = \mathbf{a}^T \mathbf{x}_k$ is at least $H$. The $H$ value, called the degree of fit of the estimated fuzzy linear model, is a value between 0 and 1 and subjectively selected by a decision maker. A physical interpretation of $H$ is that $\hat{y}_k$ is contained in the support interval of $y_k$ which has a degree of membership of at least $H$, for all $k$ (Tanaka et al., 1982). This condition can be represented as a pair of inequality constraints for each set of observations $k$, as follows:

$$\sum_{j=1}^{n} a_{mj} x_{jk} + |1 - H| \sum_{j=1}^{n} a_{sj} |x_{jk}| \geq y_k, \quad (2.4)$$

$$-\sum_{j=1}^{n} a_{mj} x_{jk} + |1 - H| \sum_{j=1}^{n} a_{sj} |x_{jk}| \geq -y_k, \quad (2.5)$$

where $x_{jk}$ is the value of the $j$th independent variable in observation $k$.

Our aim is to minimize the fuzziness in the predicted value for the dependent variable. This can be achieved by minimizing the sum of the spreads of all fuzzy estimates $a_j$, i.e., minimizing $(a_{m1} + a_{m2} + \cdots + a_{mn})$ under the restriction of Eqs. (2.4) and (2.5). This formulation represents a conventional linear program. A more general case where the membership function is nonlinear or asymmetric can be similarly accommodated, as well as differentially weighting the spreads, $a_j$'s, on the independent variables (Tanaka and Watada, 1988; Bardossy, 1990).
2.3. Model formulation

Many design tasks in practice take place in an environment in which the model components are not known precisely. One way to deal with such imprecision quantitatively is via the concept of fuzzy sets. Fuzziness can be expressed in different ways in the general model given in Eqs. (2.1)–(2.3): (i) system parameters of functional relationships \( f_i \) and \( g_j \) are fuzzy (i.e., possibilistic parameters), (ii) customers do not exhibit maximizing behavior, but rather act as satisfiers (i.e., satisficing objectives) because objective functions are fuzzy and not known precisely, and (iii) constraints are not hard, so that some leeway can be provided on the equality (or inequality) relationships (i.e., flexible constraints).

A fuzzy model can possess any combination of the above three types of fuzziness. The following sections describe how each of the above model components (system parameters, objectives, constraints) can be defined in an either crisp (non-fuzzy) or fuzzy way in QFD modeling.

2.3.1. System parameters

Fuzzy regression gives rise to a possibility distribution that accounts for the vague understanding of the observed phenomena, which is manifested by yielding fuzzy (possibilistic) parameters of the model. We obtain as a result of the fuzzy regression,

\[
y_i = \hat{f}_i(x_1, \ldots, x_n), \quad i = 1, \ldots, m, \tag{2.6}
\]

\[
x_j = \hat{g}_j(x_1, \ldots, x_{j-1}, x_{j+1}, \ldots, x_n), \quad j = 1, \ldots, n. \tag{2.7}
\]

The bar over a symbol indicates that the expression or variable is fuzzy.

For the purpose of solvability, we can convert a fuzzy equation into an equivalent system of three crisp equations through the use of the mean value and spread of a fuzzy parameter (Dubois and Prade, 1980). The crisp equations equivalent to Eqs. (2.6) and (2.7) can be constructed as:

\[
y_i = f_i(x_1, \ldots, x_n), \quad i = 1, \ldots, m, \tag{2.8}
\]

\[
y_i^L \leq f_i^L(x_1, \ldots, x_n), \quad i = 1, \ldots, m, \tag{2.9}
\]

\[
x_j = g_j(x_1, \ldots, x_{j-1}, x_{j+1}, \ldots, x_n), \quad j = 1, \ldots, n, \tag{2.10}
\]

\[
x_j^L \leq g_j^L(x_1, \ldots, x_{j-1}, x_{j+1}, \ldots, x_n), \quad j = 1, \ldots, n, \tag{2.11}
\]

\[
x_j^R \leq g_j^R(x_1, \ldots, x_{j-1}, x_{j+1}, \ldots, x_n), \quad j = 1, \ldots, n, \tag{2.12}
\]

where \( f_i, f_i^L, f_i^R \) (and \( g_j, g_j^L, g_j^R \)) are real linear vectors of mean value and left and right spreads of the estimated fuzzy parameters of \( \hat{f}_i \) (and \( \hat{g}_j \)), and \( y_i, y_i^L, y_i^R \) (and \( x_j, x_j^L, x_j^R \)) are also real linear vectors of mean value and left and right spreads of \( \hat{y}_i \) (and \( \hat{x}_j \)).

In order to build models, which employ crisp parameters, one can use the mean value estimates from fuzzy regression as the parameter estimates, and disregard the spread values. Thus, only Eqs. (2.8) and (2.11) are relevant in such a case. For models with fuzzy parameters, the whole set of Eqs. (2.8)–(2.13) needs to be employed.

2.3.2. Objective function

For a crisp objective function, we use a multi-attribute value (MAV) function, which is suitable for modeling the customer’s preferences associated with multiple performance characteristics (Keeney and Raiffa, 1976). The crisp objective function can thus be expressed as:

\[
\text{maximize } V(y_1, \ldots, y_m) \tag{2.14}
\]

where \( V(y_1, \ldots, y_m) \) is an MAV function which associates with customer satisfaction levels \( (y_1, \ldots, y_m) \) a real number. The MAV function could be additive, multiplicative, or multilinear depending upon the customer’s preference structure. For example, an additive MAV function, which is the most common type, can be expressed as (Evans et al., 1990):

\[
V(y_1, \ldots, y_m) = \sum_{i=1}^{m} w_i V_i(y_i), \tag{2.15}
\]
where (i) \( w_i \) are scaling constants representing the relative importance of performance characteristics such that \( 0 < w_i < 1, i = 1, \ldots, m \), and \( \sum_{i=1}^{m} w_i = 1 \), (ii) \( V_i(y_i) \) is an individual value function on the \( i \)th performance characteristic. \( V_i(y_i) \) is scaled in such a way that \( V_i(\text{worst } y_i) = 0 \) and \( V_i(\text{best } y_i) = 1 \) \( (i = 1, \ldots, m) \), and thus \( V(y_1, \ldots, y_m) \) is also a value between 0 and 1, with 0 being the worst and 1 the best.

To illustrate the concept, we consider a simple case where there are two performance characteristics with relative importance \((w_1, w_2) = (0.3, 0.7)\). There are also three design alternatives, \( X_1, X_2, \) and \( X_3 \), under consideration. A design alternative refers to a set of the engineering characteristic values, \( x_1, x_2, \ldots, x_n \). Suppose the individual value function \( V_1(y_1) \) is assessed as 0.1, 0.5, and 0.8 at \( X_1, X_2, \) and \( X_3 \), respectively. Then, according to Eq. (2.15), the MAV function is computed as

\[
V(y_1, y_2) = 0.3V_1(y_1) + 0.7V_2(y_2) = \begin{cases} 
0.3(0.1) + 0.7(0.4) = 0.31 & \text{at } X_1, \\
0.3(0.5) + 0.7(0.7) = 0.64 & \text{at } X_2, \\
0.3(0.8) + 0.7(0.5) = 0.59 & \text{at } X_3.
\end{cases}
\]

This means that \( X_2 \) is the best design alternative in the sense of maximizing the overall customer satisfaction level, \( V(y_1, y_2) \).

Suppose customers do not exhibit maximizing behavior, but rather act as satisfiers because the objectives are not known precisely. In such a case, the design team first establishes the aspiration levels for each performance characteristic. Let \( y_i^{\text{min}} \) and \( y_i^{\text{max}} \) respectively represent the lower and upper bounds of aspirations with respect to \( y_i \) (e.g., \( y_i^{\text{min}} = 1 \) and \( y_i^{\text{max}} = 5 \) if the five-point scale is used). Then a customer would be completely dissatisfied with a design \( X \) at which \( y_i(X) \leq y_i^{\text{min}} \), but would be completely satisfied if \( y_i(X) \geq y_i^{\text{max}} \) (From a practical viewpoint, one can think of \( y_i^{\text{max}} \) as the value for \( y_i(X) \) such that higher values of \( y_i(X) \) have little additional merit.) Here \( y_i(X) \) is the perception of the degree of achievement of the \( i \)th performance characteristic at \( X \). Thus one can express his/her satisfaction on the \( i \)th performance characteristic at \( X \) by a function, denoted as \( \mu_i(X) \), which measures how satisfactory it is that \( y_i(X) \) takes on a particular value. It can be viewed as a membership function of a fuzzy objective function. Assuming a linear form \( \mu_i(X) \), can be expressed as:

\[
\mu_i(X) = \begin{cases} 
0 & \text{if } y_i(X) \leq y_i^{\text{min}}, \\
\frac{y_i(X) - y_i^{\text{min}}}{y_i^{\text{max}} - y_i^{\text{min}}} & \text{if } y_i^{\text{min}} < y_i(X) < y_i^{\text{max}}, \\
1 & \text{if } y_i(X) \geq y_i^{\text{max}}.
\end{cases}
\]

Nonlinear membership functions for a performance characteristic can be constructed in a similar fashion (Dhingra and Moskowitz, 1991).

The model employing a fuzzy objective function attempts to optimize the overall degree of customer satisfaction derived from multiple performance characteristics. The fuzzy optimization scheme has been proven to be useful in modeling multiple criteria decision making problems involving human perception, and can be posed as an equivalent crisp optimization problem as follows (Bellman and Zadeh, 1970; Zimmermann, 1976, 1978):

Find \( x_1, x_2, \ldots, x_n \) which maximize \( \lambda \)

subject to

\[
\lambda \leq \mu_i(X), \quad i = 1, \ldots, m,
\]

where \( \lambda \) \((0 \leq \lambda \leq 1)\) represents the overall membership function value, or overall degree of satisfaction of \( m \) performance characteristics, achieved at a design \( X \). If the model has other existing constraints, they can still be used as constraints in addition to Eq. (2.18). If the performance characteristics are of varying degrees of relative priority, this can be accommodated by modifying Eq. (2.18) to:

\[
\lambda \leq \left( \mu_i(X) \right)^{w_i}, \quad i = 1, \ldots, m,
\]

where \( w_i \) is \( w_i \) (defined in Eq. (2.15)) multiplied by \( m \) (number of performance characteristics) (Yager, 1977).

2.3.3. Constraints

The functional relationships, given in Eqs. (2.8)–(2.13), may be used as strict (crisp) or
flexible (fuzzy) constraints. When the constraints are strict, the violation of any constraint by any amount renders the solution (design) infeasible. Considering the fact that, in practice, the estimated functional relationships would probably be imprecise, permitting small violations would be more realistic. This can be done by employing fuzzy (flexible) constraints. The membership function of a fuzzy constraint arising from \( S_j(X) = b_j \) is constructed as (assuming a linear form):

\[
\mu_{S_j}(X) = \begin{cases} 
0 & \text{if } S_j(X) \leq b_j - d_j \text{ or } S_j(X) \\
1 - \frac{|S_j(X) - b_j|}{d_j} & \text{if } b_j - d_j < S_j(X) \\
< b_j + d_j & \end{cases}
\]  

(2.20)

Eq. (2.20) can be easily modified for inequality constraints (e.g., \( S_j(X) \geq b_j \)).

The fuzzy constraints can be incorporated into the optimization model in the same way as the fuzzy objectives. As an example, if we allow for some flexibility in the equality constraints ((8) and (11)), the model would have

\[
\lambda_i \leq \mu_{E_i}(X, Y), \quad i = 1, \ldots, m, \\
\lambda_j \leq \mu_{G_i}(X, Y), \quad j = 1, \ldots, n,
\]  

(2.21) (2.22)

\[
y_i^L \leq f_i^L(x_1, \ldots, x_n), \quad i = 1, \ldots, m, \\
y_i^R \leq f_i^R(x_1, \ldots, x_n), \quad i = 1, \ldots, m, \\
x_j^L \leq g_j^L(x_1, \ldots, x_{j-1}, x_{j+1}, \ldots, x_n), \quad j = 1, \ldots, n, \\
x_j^R \leq g_j^R(x_1, \ldots, x_{j-1}, x_{j+1}, \ldots, x_n), \quad j = 1, \ldots, n,
\]  

(2.23)

as constraints. Eq. (2.21) is obtained by allowing some flexibility in equality constraint (8). Eq. (2.22) similarly is obtained from Eq. (2.11). The flexibility provides a design team with more design choices by widening the feasible solution set.

Eqs. (2.21)–(2.23) correspond to the constraints of a model which allows flexibility in the equality constraints, while all inequality constraints must be satisfied in a strict sense. One can easily generalize the above formulation to include flexibility in a different part of the constraint set.

2.3.4. Definition of models

We now define various crisp/fuzzy multiobjective optimization models for QFD by combining each model component described above which is suited for the design situation. We do not consider the combination of a fuzzy objective function and crisp constraints due to the lack of intuitive appeal in design situations. On the other hand, when the constraints are fuzzy, the use of a crisp objective function (i.e., determination of an extremum of a crisp function over a fuzzy domain) is not appropriate because in very rare real cases a scaling factor between the objective and constraints (and thus solution) can be found which has a real justification (Zimmermann, 1978). Therefore, this paper considers the fuzzy objective function case if and only if (at least some of) constraints are flexible. Fig. 2 shows the definition of all models. Model names consist of three letters. The first, second, and the third letter indicate whether the system parameters, objective function, and constraints are crisp (denoted by C) or fuzzy (denoted by F), respectively. For example, FCC refers to a model in which the parameters are fuzzy, while the objective function and the constraints are crisp. Moreover, Fig. 2 also shows how to formulate a specific model, e.g., FCC can be formulated by employing Eq. (2.14) as the objective function and Eqs. (2.8)–(2.13) as constraints. Because the models are defined in such a way that each model component is crisp or fuzzy, one can easily determine the value of removing or adding certain elements of fuzziness by comparing the results of the models.

3. Illustrative example and notes on applications

3.1. Illustrative example

To illustrate our methodology, we consider the following example of designing the door of an automobile (Moskowitz, 1993). Assume there are five performance characteristics which have been
scaled from 1 (worst) to 5 (best) and 6 engineering characteristics. The available data set (for customer and technical competitive analysis) is limited (the usual case in practice) and consists of only seven data points, which have been collected from the company and its 6 main competitors. The objective of the problem is to develop a new design (i.e., determine new target values for the engineering characteristics) of the door to maximize the customer satisfaction level of the company. The house of quality chart in Fig. 1 is filled in with this data. In addition to such information, the user (of the model) is required to specify the form of the membership function and the \( H \) value for fuzzy regression, the relative importance of performance characteristics, and the form of the membership function for fuzzy objectives and constraints. (See Sections 3.1.1 and 3.1.2.)

3.1.1. Parameter estimation of functional relationships

Fuzzy regression was applied to assess the \( f \) and \( g \) relations. As an example, it is presumed that the first performance characteristic, \( y_1 \), is associated with two engineering characteristics, \( x_1 \) and \( x_4 \) (first row of the body of house). For the purpose of illustration, a symmetric triangular membership function was used, where the \( H \) value was arbitrarily set to 0.5. (Selection of the membership function and \( H \) value, in an actual problem setting, should be done with care, and is formally developed in Moskowitz and Kim (1993).) Then fuzzy regression algorithm was run using the customer perceptions of the performance characteristics and current measures of the engineering characteristics as the data set. The assessed \( f \) and \( g \) relations are given in Table 1.
3.1.2. Objective function

The crisp objective function (for the models CCC, FCC) was constructed as a linear additive MAV function as follows:

\[ \text{Maximize } Z = \sum_{i=1}^{5} w_i V_i(y_i). \] (3.1)

As an illustration, we arbitrarily used \((w_1, w_2, w_3, w_4, w_5) = (0.3, 0.2, 0.1, 0.1, 0.3)\). \(V_i(y_i)\) was constructed so that condition (ii) in Eq. (2.15) is satisfied; i.e.,

\[ V_i(y_i) = 0.25y_i - 0.25, \quad i = 1, \ldots, 5. \] (3.2)

For the models with a fuzzy objective function (i.e., CFF, FFF), the information on the aspiration range of \(y_i\) (i.e., \(y_{i\text{min}}\) and \(y_{i\text{max}}\)) is required, as shown in Eq. (2.16). The range of aspiration levels can be deduced by (i) finding the solutions to the problems optimizing each individual performance characteristic, and then (ii) determining the worst and best values for each of the performance characteristics.

Five optimization problems, each of which is a single objective optimization problem with the objective of ‘Maximize \(y_i\)’ \((i = 1, \ldots, 5)\), were solved first. Then, in order to obtain the bounds on the aspiration levels for each performance characteristic, the values of \(y_i\) from the five single objective optimization problems were examined. For example, when parameters are crisp (i.e., CFF), the values of \(y_1\) at the optimal solutions of the ‘Maximize \(y_i\)’ \((i = 1, \ldots, 5)\) problems were 3.21, 1.66, 2.33, 3.21, respectively. This implies that \(y_1\), the customer perception of the degree of achievement on the first performance characteristic, would be at least 1.66 (the minimum of the five \(y_i\) values), but cannot exceed 3.21 (the maximum of the five \(y_i\) values) under the system restriction.

Thus the lower and upper bounds of the aspiration range for \(y_1\) were determined as 1.61 and 3.21, respectively. Any design with \(y_1\) less than 1.61 is unacceptable, and any design yielding \(y_1\) equal to 3.21 is totally satisfactory with respect to \(y_1\). In the same manner, the lower and upper bounds of the aspirations for other \(y_i\)’s were determined and are as follows: \((1.27, 2.30)\) for \(y_2\), \((2.53, 5.00)\) for \(y_3\), \((2.70, 4.45)\) for \(y_4\), \((1.00, 5.00)\) for \(y_5\). Similarly, the lower and upper bounds of the aspirations when the parameters are fuzzy (i.e., FFF) were determined as \((2.45, 4.45)\) for \(y_1\), \((1.81, 1.81)\) for \(y_2\), \((3.70, 3.70)\) for \(y_3\), \((2.27, 4.30)\) for \(y_4\), and \((1.35, 5.00)\) for \(y_5\). Note that the ranges are very tight for \(y_1\), \(y_2\), and \(y_3\), implying that the constraint sets are tighter when parameters are fuzzy than when the parameters are crisp, as expected. Using the ranges obtained above, the procedure outlined in Section 2.3.2 was followed to develop the membership function (\(\mu_{ yi}\) in Eq. (2.16)) and then formulate the optimization problem (Eqs. (2.17) and (2.19)).

3.1.3. Constraints

We used as constraints the fuzzy equations given in Table 1. For the models CCC and CFF

Table 1

<table>
<thead>
<tr>
<th>Intercept</th>
<th>(x_1)</th>
<th>(x_2)</th>
<th>(x_3)</th>
<th>(x_4)</th>
<th>(x_5)</th>
<th>(x_6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y_1)</td>
<td>8.11</td>
<td>-0.56</td>
<td>(0.12^*)</td>
<td>0.55</td>
<td>0.04</td>
<td></td>
</tr>
<tr>
<td>(y_2)</td>
<td>-4.04</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(y_3)</td>
<td>4.11</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(y_4)</td>
<td>5.01</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(y_5)</td>
<td>8.95</td>
<td>-0.96</td>
<td>-0.11</td>
<td>1.02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(x_1)</td>
<td>15.05</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.54</td>
</tr>
<tr>
<td>(x_2)</td>
<td>15.06</td>
<td>-0.56</td>
<td>(0.25^*)</td>
<td></td>
<td></td>
<td>0.35</td>
</tr>
<tr>
<td>(x_3)</td>
<td>2.99</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(x_4)</td>
<td>-5.03</td>
<td>-0.32</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(x_6)</td>
<td>51.00</td>
<td></td>
<td></td>
<td></td>
<td>5.17</td>
<td>(2.33^*)</td>
</tr>
</tbody>
</table>

* Numbers in parentheses represent the spreads at \(H = 0.5\).
** \(x_5\) is correlated with no other engineering characteristics. Therefore, there is no equation assessed for \(x_5\).
where no fuzziness is considered in the system parameters, we used the mean value estimates from fuzzy regression, and disregarded the spread values. For the models with fuzzy parameters (i.e., FCC, FFF), crisp equivalent constraint sets were developed from the fuzzy equations in Table 1, as described in Section 2.3.1. When the constraints are flexible (i.e., CFF, FFF), illustrative purposes, 10%, 20%, and 30% flexibility were allowed in the equality constraints, i.e., $d_j$ was set to 10%, 20%, 30% of $b_j$ in Eq. (2.20).

3.1.4. Analysis of results

Table 2 summarizes the results obtained for the various models. $Z$ represents the optimal value of the crisp MAV function, and $\lambda$ denotes the overall degree of satisfaction when a fuzzy objective function is employed. The CCC and FCC (crisp objective function) models have only $Z$ values because they are MAV function optimization problems. The CFF and FFF (fuzzy objective function) models have $\lambda$ values as well as $Z$ values. The primary concern of the fuzzy objective function models is to maximize $\lambda$, not $Z$. Hence, the $Z$ value was computed a posteriori at the $y_i$'s and $x_j$'s ($i = 1, \ldots, 5; j = 1, \ldots, 6$) that maximize the $\lambda$ value in such models. Throughout this section, we will use the $Z$ value as a performance criterion to compare the results, because $Z$ can serve as an aggregate customer satisfaction measure for either crisp or fuzzy objective function models.

3.1.4.1. Comparison of model designs vs. Current designs

In this subsection, we compare the existing designs of our example company and its competitors with those obtained by solving various models, focusing on customer perception of the design quality. First, we need to examine where our company initially stands vis-à-vis its competitors. The customer competitive analysis information contained in the house of quality in Fig. 1 indicates that our company’s product currently is weak in characteristics $y_1$, $y_2$, and $y_4$, moderate in $y_3$, and strong in $y_5$, and has the lowest value of $Z$ (= 0.42) among all seven companies. Company C has the highest $Z$ of 0.64, and companies D and F also have high $Z$ values of 0.57 and 0.58, respectively.

The $Z$ value of the design from the CCC model (= 0.61) is much higher than our current $Z$ value (= 0.42) and is comparable to that of company C (= 0.64). (In this example, Company C’s product has a higher $Z$ value than the design from the CCC model. This could happen because the optimiza-

<table>
<thead>
<tr>
<th>Models</th>
<th>Characteristics</th>
<th>CCC</th>
<th>CFF</th>
<th>FCC</th>
<th>FFF</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>10%</td>
<td>20%</td>
<td>30%</td>
<td>10%</td>
</tr>
<tr>
<td>$y_1$</td>
<td></td>
<td>3.21</td>
<td>3.50</td>
<td>3.68</td>
<td>4.10</td>
</tr>
<tr>
<td>$y_2$</td>
<td></td>
<td>2.30</td>
<td>2.89</td>
<td>3.30</td>
<td>2.38</td>
</tr>
<tr>
<td>$y_3$</td>
<td></td>
<td>2.53</td>
<td>3.58</td>
<td>3.81</td>
<td>3.96</td>
</tr>
<tr>
<td>$y_4$</td>
<td></td>
<td>2.70</td>
<td>3.64</td>
<td>3.98</td>
<td>3.72</td>
</tr>
<tr>
<td>$y_5$</td>
<td></td>
<td>5.00</td>
<td>4.35</td>
<td>4.50</td>
<td>4.86</td>
</tr>
<tr>
<td>$x_1$</td>
<td></td>
<td>8.83</td>
<td>8.43</td>
<td>8.00</td>
<td>8.33</td>
</tr>
<tr>
<td>$x_2$</td>
<td></td>
<td>12.52</td>
<td>14.40</td>
<td>14.02</td>
<td>12.83</td>
</tr>
<tr>
<td>$x_3$</td>
<td></td>
<td>6.87</td>
<td>7.25</td>
<td>7.45</td>
<td>6.75</td>
</tr>
<tr>
<td>$x_4$</td>
<td></td>
<td>1.00</td>
<td>2.13</td>
<td>2.23</td>
<td>2.40</td>
</tr>
<tr>
<td>$x_5$</td>
<td></td>
<td>6.45</td>
<td>5.87</td>
<td>5.64</td>
<td>6.04</td>
</tr>
<tr>
<td>$x_6$</td>
<td></td>
<td>52.09</td>
<td>56.61</td>
<td>55.80</td>
<td>57.08</td>
</tr>
<tr>
<td>$Z$</td>
<td></td>
<td>0.61</td>
<td>0.66</td>
<td>0.72</td>
<td>0.73</td>
</tr>
<tr>
<td>$\lambda$</td>
<td></td>
<td>–</td>
<td>0.65</td>
<td>0.72</td>
<td>0.76</td>
</tr>
</tbody>
</table>

* Flexibility permitted.
tion uses as constraints the functional relationships assessed from the benchmarking data set, which are not perfect. Thus it would be more sensible to permit some flexibility in the constraints to obtain more realistic results.) Compared with our current design, the CCC model improved $y_1$, $y_2$, $y_5$ by trading off $y_3$ and $y_4$, which are the least important performance characteristics. The engineering characteristic values were determined to achieve such a value tradeoff in the most efficient way. For example, CCC yielded a significantly higher $y_1$ than in our current design. The value of $y_3$ is negatively and positively correlated with $x_1$ and $x_4$, respectively (Table 1). In order to increase $y_1$, the resulting design lowered the level of $x_1 \ (i.e., \ from \ 11 \ to \ 8.83)$. Engineering characteristic $x_4$ did not increase in this example (in fact, $x_4$ decreased from 3 to 1) because the correlation between $y_1$ and $x_4$ is rather weak and making $x_4$ larger would lower the satisfaction on the other most important characteristic ($y_3$) significantly.

The model considers all such interactions between performance characteristics and engineering characteristics as well as those among the engineering characteristics simultaneously, and determines the optimal engineering characteristic levels.

The FCC model yielded a design with a $Z$ value of 0.56, which is comparable to those of companies C, D, and F. FCC achieved the highest possible value of $y_5$ while maintaining moderate levels for other performance characteristics.

CFF with 10% of flexibility resulted in a significantly better design than CCC or any existing company in terms of customer satisfaction. The designs from FFF also achieved very high $\lambda$ values, while the $Z$ values with 10% and 20% flexibility are comparable to those of companies C, D, and F.

3.1.4.2. Comparison of customer satisfaction: Effects of possibility and flexibility:. In this subsection, we examine the impact of using fuzzy (possibilistic) parameters and fuzzy (flexible) constraints on design by comparing the results of the various models (Fig. 3). Such a comparison can be viewed from an information value perspective, in the sense that it suggests how much one might be willing to pay to eliminate possibility or add flexibility. In Fig. 3, a horizontal and a vertical arrow indicates the addition of possibility and flexibility into the model, respectively. $\Delta Z$ recorded along the arc represents the change in the $Z$ value as a result of adding possibility and/or flexibility.

Comparison of CCC and FCC shows that inherent fuzziness (possibility) in the system parameters, as expected, has a negative effect on the $Z$ value. The CCC model yielded the $Z$ value of 0.61 by assuming the system equations are crisp. However, the actually attainable $Z$ value was only 0.56 when fuzziness in the system parameters was explicitly considered in the FCC model. This implies that if a designer does not consider the inherent possibility when it actually exists, then the quality of the target design ($Z$) is overestimated (e.g., $\Delta Z = Z(FCC) - Z(CCC) = 0.56 - 0.61 = -0.05$, 9% of $Z(FCC)$), and the design per se may not even be feasible.

Comparison of the CCC ($Z=0.61$) and CFF ($Z=0.66, 0.72, 0.73$, and $\lambda = 0.65, 0.72, 0.76$ when 10%, 20%, 30% flexibility was allowed, respectively) models reveals that the CFF model achieves a higher level of customer satisfaction ($Z$) owing to the flexibility allowed in the constraints. As more flexibility was allowed, the $Z$ value also increased in our example, although there is no such guarantee on the improvement of the $Z$ value (because the objective of the CFF is to maximize $\lambda$).

Realistically, the assessed functional relationships are virtually never precise. It would be reasonable to relax the system equations in some way to compensate for the impreciseness of the relationships. Without allowing such flexibility, the designer cannot achieve what is realistically attainable (e.g., $\Delta Z = Z(CFF) - Z(CCC) = 0.12$, a 20% increase in customer satisfaction, when 30% flexibility was allowed).

Note also that in the CFF model, the overall degree of satisfaction with respect to the $\lambda$ value increased with flexibility allowed. However, the marginal rate of increase diminished as more flexibility was allowed. The increase of the flexibility from 10% to 20% improved the $\lambda$ value by 0.07 (= 0.72 – 0.65) or by 11% while the incremental improvement from 20% to 30% flexibility was only 0.04 (= 0.76 – 0.72), which is just a 6% improvement. The comparison of FCC and FFF shows essentially the same pattern with differing
magnitude. However, too much flexibility would deteriorate the validity of the system description. There certainly has to be a limit on the amount of flexibility to be permitted, which needs to be determined considering the specific design situation. The limit would depend on such factors as tolerances on engineering characteristics, credibility of the assessed system equations, feasibility of technological innovation.

The combined effects of possibility and flexibility can be presented by comparing the results of the CCC and the FFF problem. The changes in the customer satisfaction level \(Z\) in this case could be negative or positive depending on the impact of having possibility and/or the amount of flexibility allowed; i.e., \(\Delta Z = Z(\text{FFF}) - Z(\text{CCC}) = -0.01, 0.02, 0.08\) when 10%, 20%, 30% flexibility was allowed, respectively.

### 3.2. Notes on applications

A novice-friendly decision support system prototype for QFD, called QFD Optimizer, has been developed to implement the modeling approach described in Section 2. (See Moskowitz and Kim, 1997, for details.) We have used QFD Optimizer successfully in various customized, corporate specific executive education programs involving designing a bottle and cap for a pharmaceutical product.
product (Moskowitz and Ward, 1998), as well as in designing a manufacturing strategy for a new consumer electronics plant being located in Mexico (Moskowitz, 1998).

We have found that eliciting information, such as performance characteristics and customer perceptions posed no major difficulties, at least in the applications mentioned above. Using a facilitator and/or structured questionnaire worked well and yielded consistent and credible responses.

4. Conclusions

An integrated, fuzzy theoretic approach to formulating and solving the QFD problem has been presented. Multiattribute value theory combined with fuzzy regression and fuzzy optimization theory could allow the design team to mathematically consider tradeoffs among the various performance characteristics and the inherent fuzziness in the system. In addition, the modeling approach presented makes it possible to observe separately as well as conjointly the effects of possibility and flexibility on an overall design. Knowledge of the impact of the possibility and flexibility on customer satisfaction can also serve as a guideline for acquiring additional information to reduce fuzziness in the system parameters or determining how much flexibility is warranted or possible to improve a design. The fuzzy multiobjective models developed and illustrated may be applied to a wide variety of product design problems where multiple design criteria and system functional relationships are interdependent and conflicting in an uncertain, qualitative, and fuzzy way.

Our modeling approach also suggests directions for future research. The basic model presented could be expanded to allow extended linkages among the houses of quality across the design/production chain. For example, engineering characteristics like pound per foot of door seal resistance can act as a criterion to be optimized in a parts deployment house—a house of quality at the next level in the chain. Modeling several houses simultaneously in a single framework could presumably aid in conveying the voice of the customer to manufacturing.

Another research direction would be to use a more refined methodology in our model. The system equations $f$ and $g$ which represent the relationships among performance characteristics and engineering characteristics, and among engineering characteristics, were assessed in a linear functional form. This linearity may be a rather strict assumption in real design problems. It would be of value to develop the cases that can utilize various sources of assessing relationships such as simulation, design of experiments, or knowledge coming from physical relationships.

It has been discussed that fuzziness (possibility) of the system parameters deteriorates the optimal design, while flexibility improves it. Once the solution to an optimization problem is obtained, the designer may wish to employ sensitivity analysis to examine the effect of possibility and/or flexibility of individual fuzzy parameters and constraints respectively to determine which parameters or constraints should be targeted to improve system design most efficiently. It would be useful to develop a systematic procedure for identifying such parameters or constraints which have the greatest impact on system design performance.

Acknowledgements

This research was supported in part by the research fund from Pohang University of Science and Technology (1RB9703101 and 1RB98004), Ministry of Science and Technology of Korea (through STEPI), the Purdue University Krannert School of Management’s Center for the Management of Manufacturing Enterprises, the Purdue University Engineering Research Center for Intelligent Manufacturing Systems under NSF Grant CDR 850002, and NSF Grant INT 9114147.

References


Clausing, D., 1994. Total quality development, ASME Press, Dover, NH.


Moskowitz, H., 1993. Fjorde motor company: Revitalizing product development, Krannert Graduate School of Management, Purdue University, West Lafayette, IN.


