Unit 2 – RDF Formal Semantics in Detail

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Where are we?

- Last time we learnt:
  - Basic ideas about RDF and how it is published
  - Turtle Syntax for RDF - we know how to write RDF
  - Basic SPARQL queries - we (roughly) know how to query RDF
  - Overview of RDF Schema & OWL
- Today and on Monday:
  - RDF formal semantics...
  - ... which will be the basis for SPARQL's formal semantics
  - ... and also for RDF Schema & OWL
- RDF Schema semantics & SPARQL semantics
- Also on Monday:
  - Discussion of Assignment 1 (plus Assignment2)
  - Some initial suggestions for final presentation topics

Unit Outline

1. Semantics of RDF+RDFS
2. RDF Graph – Formal Definitions
3. RDF Interpretations and Simple Entailment
4. APPENDIX: Simple RDF Entailment is NP-complete

The Semantics of RDF graphs:

```
@prefix rdfs: <http://www.w3.org/2000/01/rdf-schema#> .
@prefix rdf: <http://www.w3.org/1999/02/22-rdf-syntax-ns#> .
@prefix foaf: <http://xmlns.com/foaf/0.1/> .
<http://www.mat.unical.it/~ianni/foaf.rdf> a foaf:PersonalProfileDocument.
<http://www.mat.unical.it/~ianni/foaf.rdf> foaf:maker _:me .
<http://www.mat.unical.it/~ianni/foaf.rdf> foaf:primaryTopic _:me .
:me a foaf:Person .
:me foaf:name "Giovambattista Ianni" .
:me foaf:homepage <http://www.gibbi.com> .
:me foaf:phone <tel:+39-0984-496430> .
:me foaf:knows [ a foaf:Person ;
                 foaf:name "Wolfgang Faber" ;
                 rdfs:seeAlso <http://www.kr.tuwien.ac.at/staff/faber/foaf.rdf>].
:me foaf:knows [ a foaf:Person .
                 foaf:name "Axel Polleres" ;
                 rdfs:seeAlso <http://www.polleres.net/foaf.rdf>].
:me foaf:knows [ a foaf:Person .
                 foaf:name "Thomas Eiter" ] .
:me foaf:knows [ a foaf:Person .
                 foaf:name "Alessandra Martello" ] .
```

The Semantics of RDF graphs:

Recall from last time:
Each RDF graph can – essentially – be viewed as a first-order formula:

$$
\begin{array}{l}
\exists b1, b2, b3, b4 \\
(triple(\texttt{foaf.rdf}, \texttt{rdf:type}, \texttt{PersonalProfileDocument}) \\
\ \wedge\ triple(\texttt{foaf.rdf}, \texttt{maker}, me) \\
\ \wedge\ triple(\texttt{foaf.rdf}, \texttt{primaryTopic}, me) \\
\ \wedge\ triple(me, \texttt{rdf:type}, \texttt{Person}) \\
\ \wedge\ triple(me, \texttt{name}, \texttt{"Giovambattista Ianni"}) \\
\ \wedge\ triple(me, \texttt{homepage}, \texttt{http://www.gibbi.com}) \\
\ \wedge\ triple(me, \texttt{phone}, \texttt{tel:+39-0984-496430}) \\
\ \wedge\ triple(me, \texttt{knows}, b2) \wedge\ triple(b1, \texttt{type}, \texttt{Person}) \\
\ \wedge\ triple(b1, \texttt{name}, \texttt{"Wolfgang Faber"}) \\
\ \wedge\ triple(b1, \texttt{rdfs:seeAlso}, \texttt{http://www.kr.tuwien...}) \\
\ \wedge\ triple(me, \texttt{knows}, b1) \wedge\ triple(b1, \texttt{rdf:type}, \texttt{Person}) \\
\ \wedge\ triple(b2, \texttt{name}, \texttt{"Axel Polleres"}) \\
\ \wedge\ triple(b2, \texttt{rdfs:seeAlso}, \texttt{http://www.polleres...}) \\
\ \wedge\ triple(me, \texttt{knows}, b3) \wedge\ triple(b1, \texttt{rdf:type}, \texttt{Person}) \\
\ \wedge\ triple(b3, \texttt{name}, \texttt{"Thomas Eiter"}) \\
\ \wedge\ triple(me, \texttt{knows}, b4) \wedge\ triple(b1, \texttt{type}, \texttt{Person}) \\
\ \wedge\ triple(b4, \texttt{name}, \texttt{"Alessandra Martello"}))
\end{array}
$$

The Semantics of the RDFS vocabulary:

The formal semantics of RDF(S) [Hayes, 2004] is accompanied by a set of (informative) entailment rules . . . can be written down as the following first-order formulas:

$$\forall S,P,O\ (triple(S,P,O) \supset triple(S,\texttt{rdf:type},\texttt{rdfs:Resource}))$$
$$\forall S,P,O\ (triple(S,P,O) \supset triple(P,\texttt{rdf:type},\texttt{rdf:Property}))$$
$$\forall S,P,O\ (triple(S,P,O) \supset triple(O,\texttt{rdf:type},\texttt{rdfs:Resource}))$$
$$\forall S,P,O\ (triple(S,P,O) \land triple(P,\texttt{rdfs:domain},C) \supset triple(S,\texttt{rdf:type},C))$$
$$\forall S,P,O,C\ (triple(S,P,O) \land triple(P,\texttt{rdfs:range},C) \supset triple(O,\texttt{rdf:type},C))$$
$$\forall C\ (triple(C,\texttt{rdf:type},\texttt{rdfs:Class}) \supset triple(C,\texttt{rdfs:subClassOf},\texttt{rdfs:Resource}))$$
$$\forall C_1,C_2,C_3\ (triple(C_1,\texttt{rdfs:subClassOf},C_2) \land triple(C_2,\texttt{rdfs:subClassOf},C_3) \supset triple(C_1,\texttt{rdfs:subClassOf},C_3))$$
$$\forall S,C_1,C_2\ (triple(S,\texttt{rdf:type},C_1) \land triple(C_1,\texttt{rdfs:subClassOf},C_2) \supset triple(S,\texttt{rdf:type},C_2))$$
$$\forall S,C\ (triple(S,\texttt{rdf:type},C) \supset triple(C,\texttt{rdf:type},\texttt{rdfs:Class}))$$
$$\forall C\ (triple(C,\texttt{rdf:type},\texttt{rdfs:Class}) \supset triple(C,\texttt{rdfs:subClassOf},C))$$
$$\forall P_1,P_2,P_3\ (triple(P_1,\texttt{rdfs:subPropertyOf},P_2) \land triple(P_2,\texttt{rdfs:subPropertyOf},P_3) \supset triple(P_1,\texttt{rdfs:subPropertyOf},P_3))$$
$$\forall S,P_1,P_2,O\ (triple(S,P_1,O) \land triple(P_1,\texttt{rdfs:subPropertyOf},P_2) \supset triple(S,P_2,O))$$
$$\forall P\ (triple(P,\texttt{rdf:type},\texttt{rdf:Property}) \supset triple(P,\texttt{rdfs:subPropertyOf},P))$$

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$\forall S, P, O\ (triple(S, P, O) \supset triple(O, \texttt{rdf:type}, \texttt{rdfs:Resource}))$

$\forall S, P, O\ (triple(S, P, O) \wedge triple(P, \texttt{rdfs:domain}, C) \supset triple(S, \texttt{rdf:type}, C))$

$\forall S, P, O, C\ (triple(S, P, O) \wedge triple(P, \texttt{rdfs:range}, C) \supset triple(O, \texttt{rdf:type}, C))$

$\forall C\ (triple(C, \texttt{rdf:type}, \texttt{rdfs:Class}) \supset triple(C, \texttt{rdfs:subClassOf}, \texttt{rdfs:Resource}))$

$\forall C_1, C_2, C_3\ (triple(C_1, \texttt{rdfs:subClassOf}, C_2) \wedge$
$\quad triple(C_2, \texttt{rdfs:subClassOf}, C_3) \supset triple(C_1, \texttt{rdfs:subClassOf}, C_3))$

$\forall S, C_1, C_2\ (triple(S, \texttt{rdf:type}, C_1) \wedge triple(C_1, \texttt{rdfs:subClassOf}, C_2) \supset triple(S, \texttt{rdf:type}, C_2))$

$\forall S, C\ (triple(S, \texttt{rdf:type}, C) \supset triple(C, \texttt{rdf:type}, \texttt{rdfs:Class}))$

$\forall C\ (triple(C, \texttt{rdf:type}, \texttt{rdfs:Class}) \supset triple(C, \texttt{rdfs:subClassOf}, C))$

$\forall P_1, P_2, P_3\ (triple(P_1, \texttt{rdfs:subPropertyOf}, P_2) \wedge$
$\quad triple(P_2, \texttt{rdfs:subPropertyOf}, P_3) \supset triple(P_1, \texttt{rdfs:subPropertyOf}, P_3))$

$\forall S, P_1, P_2, O\ (triple(S, P_1, O) \wedge triple(P_1, \texttt{rdfs:subPropertyOf}, P_2) \supset triple(S, P_2, O))$

$\forall P\ (triple(P, \texttt{rdf:type}, \texttt{rdf:Property}) \supset triple(P, \texttt{rdfs:subPropertyOf}, P))$

RDFS Semantics Example: The FOAF ontology

FOAF Ontology:

- *Each* `Person` *is a* `Agent` (subclass)
- *The* `img` *property is more specific than* `depiction` (subproperty)
- `img` *is a relation between* `Persons` *and* `Imgages` (domain/range)
- `knows` *is a relation between two* `Persons` (domain/range)

⋮

RDFS: Semantics

⋮

$$\forall S, C_1, C_2\ (triple(S, \texttt{rdf:type}, C_1)\ \wedge triple(C_1, \texttt{rdfs:subClassOf}, C_2) \supset triple(S, \texttt{rdf:type}, C_2))$$

⋮

Data:

```
:me rdf:type foaf:Person .
```

RDFS Semantics Example: The FOAF ontology

FOAF Ontology in RDF:

- `foaf:Person rdfs:subClassOf foaf:Agent .`
- `foaf:img rdfs:subPropertyOf foaf:depiction .`
- `foaf:img rdfs:domain foaf:Person ; rdfs:range foaf:Image .`
- `foaf:knows rdfs:domain foaf:Person ; rdfs:range foaf:Person .`

⋮

RDFS: Semantics

⋮

$$\forall S, C_1, C_2\ (triple(S, \texttt{rdf:type}, C_1) \wedge triple(C_1, \texttt{rdfs:subClassOf}, C_2) \supset triple(S, \texttt{rdf:type}, C_2))$$

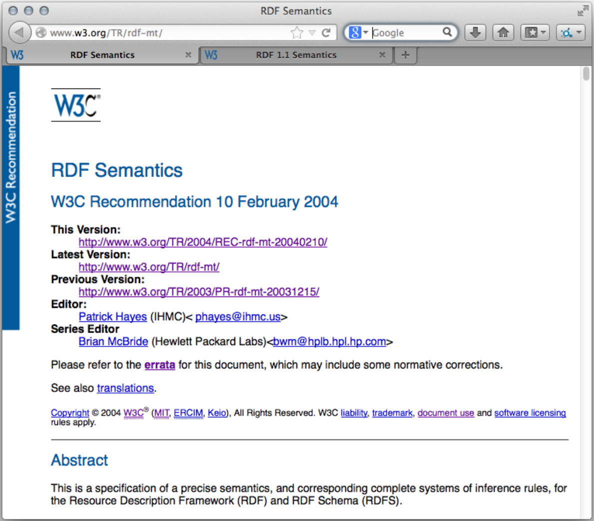
⋮

Data:

```
:me rdf:type foaf:Person .
:me rdf:type foaf:Agent .
```

RDF + RDFS Semantics according to W3C:

http://www.w3.org/TR/rdf-mt/


RDF Semantics

www.w3.org/TR/rdf-mt/

W3C Recommendation

RDF Semantics

W3C Recommendation 10 February 2004

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Please refer to the **errata** for this document, which may include some normative corrections.

See also translations.

Abstract

This is a specification of a precise semantics, and corresponding complete systems of inference rules, for the Resource Description Framework (RDF) and RDF Schema (RDFS).

RDF + RDFS Semantics according to W3C:

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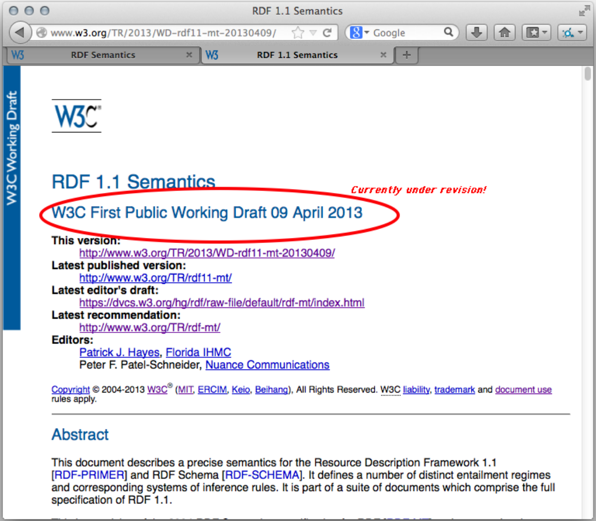

RDF 1.1 Semantics

W3C First Public Working Draft 09 April 2013

This version:
http://www.w3.org/TR/2013/WD-rdf11-mt-20130409/
Latest published version:
http://www.w3.org/TR/rdf11-mt/
Latest editor's draft:
https://dvcs.w3.org/hg/rdf/raw-file/default/rdf-mt/index.html
Latest recommendation:
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Abstract

This document describes a precise semantics for the Resource Description Framework 1.1 [RDF-PRIMER] and RDF Schema [RDF-SCHEMA]. It defines a number of distinct entailment regimes and corresponding systems of inference rules. It is part of a suite of documents which comprise the full specification of RDF 1.1.

Unit Outline

RDF Graph – Formal Definitions

Let U be the set of URIs, B be the set of blank nodes (or "variables"), $L = L_t \cup L_p \cup L_{lang}$ be the set of literals (i.e., typed, plain, and plain lang-tagged)

An RDF graph, or simply a graph, is a set of RDF triples from $UB \times U \times UBL$.[1]

A vocabulary of a graph V_G is the subset of UL mentioned in the graph.

A graph or triple without blank nodes is also called ground

[1] We write short e.g. UBL for $U \cup B \cup L$.

RDF Graph – Example 1

Node: "edge labels" may appear as nodes and vice versa, e.g.

G_1 :

```
ex:alice foaf:knows ex:bob.
ex:alice foaf:name "Alice".
foaf:knows rdfs:domain foaf:Person.
```

G_2 :

```
ex:alice rdf:type foaf:Person.
```

G_3 :

```
_:alice foaf:knows ex:bob.
_:alice foaf:name _:name.
```

G_4 :

```
_:alice foaf:knows ex:bob.
_:alice foaf:name _:alice.
```

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G_2 :

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```

G_3 :

$Alice$ `foaf:knows ex:bob.`
$Alice$ `foaf:name` $Name$.

G_4 :

$Alice$ `foaf:knows ex:bob.`
$Alice$ `foaf:name` $Alice$.

Again, we will occasionally write blank nodes as like this Var, to make clearer that actually they ammount to existentially quantified variables.

RDF Graph – Example 2

That is also a valid RDF graph:

G_5 :

```
rdfs:Resource rdf:type rdfs:Class.
rdf:Property rdf:type rdfs:Resource.
rdf:Property rdfs:subclassOf rdfs:Resource.
rdf:Property rdf:type rdfs:Class.
rdfs:Class rdf:type rdfs:Resource.
rdfs:Class rdf:type rdfs:Class.
rdfs:Class rdfs:subclassOf rdfs:Resource.
rdfs:Class rdfs:subclassOf rdfs:Class.
```

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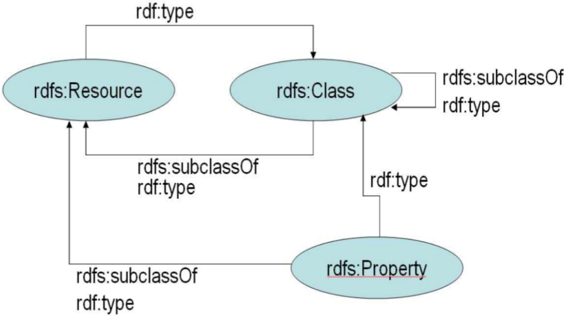

RDF Graph – Example 3

Or that:

G_6 :

```
rdfs:subClassOf rdfs:subPropertyOf rdfs:Resource.
rdfs:subClassOf rdfs:subPropertyOf rdfs:subPropertyOf.
rdf:type rdfs:subPropertyOf rdfs:subClassOf.
rdfs:subClassOf rdf:type owl:SymmetricProperty.
```

Definitions

Assume a blank node mapping $\mu : B \to UBL$.

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By $\mu(G)$ we denote the graph obtained from G by replacing each blank node x with $\mu(x)$.

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An RDF graph is *lean* if it has no instance which is a proper subgraph of the graph. Non-lean graphs have internal redundancy and express the same content as their lean subgraphs.

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Two graphs which differ only in the identity of their blank nodes, are considered to be *equivalent*.

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The *merge* of a set of graphs is obtained by renaming ("standardize apart") blank nodes in each graph such that no blank nodes between any two graphs are in common and then taking the union of all triples, we write $G1 \uplus G2$ for the graph merge between two graphs $G1, G2$.

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Questions

What is meant by "redundancy" and "same content"?

Graph Merge: Example

G_7 :

```
_:x foaf:knows ex:bob.
_:x foaf:knows _:y.
```

G_8 :

```
_:x foaf:knows ex:bob.
_:x foaf:knows _:x.
```

$G_7 \uplus G_8$: ???

Graph Merge: Example

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_:x foaf:knows _:x.
```

$G_7 \uplus G_8$:

```
_:x foaf:knows ex:bob.
_:x foaf:knows _:y.
_:z foaf:knows ex:bob.
_:z foaf:knows _:z.
```

Lean and non-lean graphs: Examples

G_7 : non-lean

```
_:x foaf:knows ex:bob.
_:x foaf:knows _:y.
```

G_8 : lean

```
_:x foaf:knows ex:bob.
_:x foaf:knows _:x.
```

Why?

Lean and non-lean graphs: Examples

G_7 : non-lean

$\exists x, y. triple(x, \texttt{knows}, \texttt{bob}) \wedge triple(x, \texttt{knows}, y)$

G_8 : lean

$\exists x. triple(x, \texttt{knows}, \texttt{bob}) \wedge triple(x, \texttt{knows}, x)$

Becomes clear if we look at first-order "reading" of the RDF graph, where we treat blank nodes as existential variables and triples in a predicate $triple$. With this reading, one could say: $G'_7 = \{\texttt{\_:x foaf:knows ex:bob.}\} \models G_7$

Lean and non-lean graphs: Examples

G_7 : non-lean

$\exists x.triple(x, \texttt{knows}, \texttt{bob}) \models$
$\exists x, y.triple(x, \texttt{knows}, \texttt{bob}) \land triple(x, \texttt{knows}, y)$

G_8 : lean

$\exists x.triple(x, \texttt{knows}, \texttt{bob}) \not\models$
$\exists x.triple(x, \texttt{knows}, \texttt{bob}) \land triple(x, \texttt{knows}, x)$

Becomes clear if we look at first-order "reading" of the RDF graph, where we treat blank nodes as existential variables and triples in a predicate $triple$. With this reading, one could say: $G'_7 = \{\texttt{\_:x foaf:knows ex:bob.}\} \models G_7$

We use first-order *entailment* here. Entailment is typically defined in terms of a model theory (interpretation, satisfaction, models)...

RDF has its own model theory!

Unit Outline

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3. RDF Interpretations and Simple Entailment
4. APPENDIX: Simple RDF Entailment is NP-complete

Model theoretic semantics – in general

A model theory is usually defined using the following "components":

- Defining a notion of an interpretation I, consisting of separate interpretation functions
  - i.e., defining how are constants, variables and logical conectives, formulas being "interpreted" in a possible real world.
- A satisfaction relation between interpretations and theories (in our case graphs), written $I \models G$, which says:
  - I is an interpretation satisfying G, or I is a model of G
- An entailment relation between theories (in our case graphs), written $G \models G'$, which says
  - all models of G are also models of G'

Simple Interpretations 1/4

"*interpretation* I*: ... i.e. how are constants, variables, predicates, formulas being "interpreted" in a possible real world.*"

What does that mean for RDF?

- RDF "constants" ... subjects, objects, i.e. UL
- RDF "variables" ... blank nodes, i.e. B
- RDF "predicates" ... predicates, i.e. U
- RDF "formulas" ... triples, graphs.

Simple Interpretations 1/4

"*interpretation* I*: . . . i.e. how are constants, variables, predicates, formulas being "interpreted" in a possible real world.*"

What does that mean for RDF?

- RDF "constants" . . . subjects, objects, i.e. UL
- RDF "variables" . . . blank nodes, i.e. B
- RDF "predicates" . . . predicates, i.e. U
- RDF "formulas" . . . triples, graphs.

Now here we have something unlike classical logic... URIs can actually need to be interpreted "as predicates" or "as constants" depending on where they appear in the graph.

To cater for that, RDF defines a very general notion of interpretation.

Simple Interpretations 2/4

A simple interpretation I over vocabulary V is a 6-tuple $I = \langle IR, IP, IEXT, IS, IL, LV \rangle$, s.t.

1. A non-empty set IR of resources.
2. A set IP, called the set of properties,
3. A mapping $IEXT : IP \rightarrow 2^{(IR \times IR)}$, i.e. assigns a set of pairs $\langle x, y \rangle$ with $x, y \in IR$.
4. A mapping $IS : U \cap V \rightarrow IR \cup IP$
5. A mapping $IL : L_t \cap V$ into IR.
6. A distinguished subset $LV \subset IR$, called the set of literal values, which contains all the plain literals in V, i.e. $LV \subseteq L_p \cup L_{lang}$.

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1. A non-empty set IR of resources.
   - *called the domain or universe of I*
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   - *not necessaily disjoint with IR*
3. A mapping $IEXT : IP \to 2^{(IR \times IR)}$, i.e. assigns a set of pairs $\langle x, y \rangle$ with $x, y \in IR$.
   - *intuitivelty, assigns a binary relation between subjects and objects to properties.*
4. A mapping $IS : U \cap V \to IR \cup IP$
   - *this basically says, URIs can be both constants and predicates*
5. A mapping $IL : L_t \cap V$ into IR.
   - *typed literals are constants.*
6. A distinguished subset $LV \subset IR$, called the set of literal values, which contains all the plain literals in V, i.e. $LV \subseteq L_p \cup L_{lang}$.
   - *plain literals in RDF are special, they are always interpreted as themselves*

Simple Interpretations 2/4

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4. A mapping $IS : U \cap V \rightarrow IR \cup IP$
   - *this basically says, URIs can be both constants and predicates*
5. A mapping $IL : L_t \cap V$ into IR.
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6. A distinguished subset $LV \subset IR$, called the set of literal values, which contains all the plain literals in V, i.e. $LV \subseteq L_p \cup L_{lang}$.
   - *plain literals in RDF are special, they are always interpreted as themselves*

Simple Interpretations 2/4

A simple interpretation I over vocabulary V is a 6-tuple $I = \langle IR, IP, IEXT, IS, IL, LV \rangle$, s.t.

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2. A set IP, called the set of properties,
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Simple Interpretations 3/4

Interpreting ground graphs (i.e. without blank nodes):

- Interpreting constants:
  - if $e = $ "aaa" $\in V \cap L_p$, then $I(e) = aaa \in LV$
  - if $e = $ "aaa"@ttt $\in V \cap L_{lang}$, then $I(e) = < aaa, ttt > \in LV$
  - if $e \in V \cap L_t$, then $I(e) = IL(e)$
  - if $e \in V \cap U$, then $I(e) = IS(e)$

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- Interpreting ground triples:
  - if $t = \texttt{s p o.}$, is a ground triple, then
    - $I(t) = true$ if $\texttt{s}, \texttt{p}, \texttt{o} \in V \land I(\texttt{p}) \in IP \land \langle I(\texttt{s}), I(\texttt{o}) \rangle \in IEXT(I(\texttt{p}))$
    - $I(t) = false$, otherwise

Simple Interpretations 3/4

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  - if G is a ground RDF graph then $I(G) = true$ if and only if $I(t) = true$ for all triples $t \in G$, .

Simple Interpretations 3/4

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- Interpreting ground graphs:
  - if G is a ground RDF graph then $I(G) = true$ if and only if $I(t) = true$ for all triples $t \in G$, .

Satisfaction

If $I(G) = true$ we also say I satisfies G, written $I \models G$

Simple Interpretation – Example ground graphs

Take the following artificial vocabulary:
$\{\texttt{ex : a}, \texttt{ex : b}, \texttt{ex : c}, "\texttt{whatever}", "\texttt{whatever}"\hat{}\hat{}\texttt{ex : b}\}$

$IR = LV \cup \{1, 2\}$

$IP = \{1\}$

$IEXT(1) = \{< 1, 2 >, < 2, 1 >\}$

$IS(\texttt{ex : a}) = IS(\texttt{ex : b}) = 1, IS(\texttt{ex : c}) = 2$

$IL("\texttt{whatever}"\hat{}\hat{}\texttt{ex : b}) = 2$

Simple Interpretation – Example ground graphs

Take the following artificial vocabulary:
$\{\texttt{ex:a}, \texttt{ex:b}, \texttt{ex:c}, \texttt{"whatever"}, \texttt{"whatever"}\hat{}\hat{}\texttt{ex:b}\}$
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G_9 :

```
ex:a ex:b ex:c .
ex:c ex:a ex:a .
ex:c ex:b ex:a .
ex:a ex:b "whatever"^^ex:b .
```

$I(G_9) = true$, i.e., $I \models G_9$:

Simple Interpretation – Example ground graphs

Take the following artificial vocabulary:
$\{\texttt{ex:a}, \texttt{ex:b}, \texttt{ex:c}, \texttt{"whatever"}, \texttt{"whatever"}\hat{}\hat{}\texttt{ex:b}\}$

$IR = LV \cup \{1, 2\}$

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$IS(\texttt{ex:a}) = IS(\texttt{ex:b}) = 1, IS(\texttt{ex:c}) = 2$

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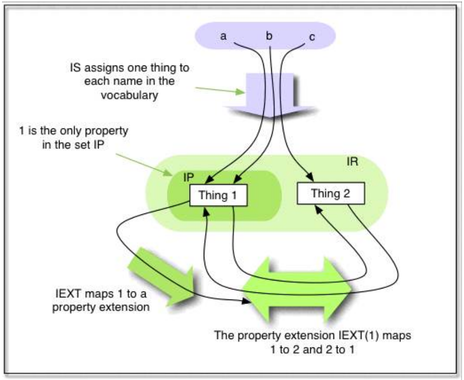

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ex:a ex:b ex:c .
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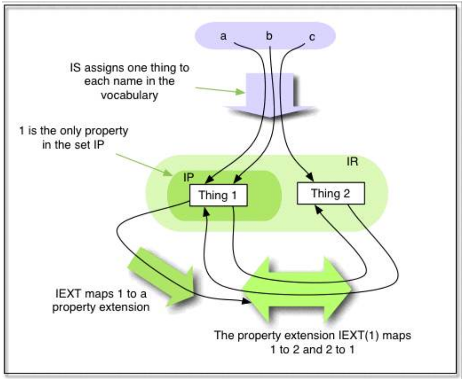

$I(G'_9) = \mathit{false}$, i.e., I doesn't satisfy **any** triple in G'_9:

Simple Interpretation – Example ground graphs

Take the following artificial vocabulary:
$\{\texttt{ex:a}, \texttt{ex:b}, \texttt{ex:c}, \texttt{"whatever"}, \texttt{"whatever"}\hat{}\hat{}\texttt{ex:b}\}$
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$IS(\texttt{ex:a}) = IS(\texttt{ex:b}) = 1, IS(\texttt{ex:c}) = 2$

$IL(\texttt{"whatever"}\hat{}\hat{}\texttt{ex:b}) = 2$

G'_9 :

`ex:a ex:c ex:b .`	$IS(\texttt{ex:c}) = 2 \notin IP$
`ex:a ex:b ex:b .`	$\langle 1, 1\rangle \notin IEXT(IS(\texttt{ex:b}))$
`ex:c ex:b ex:c .`	$\langle 2, 2\rangle \notin IEXT(IS(\texttt{ex:b}))$
`ex:a ex:b "whatever".`	$\langle 1, "whatever"\rangle \notin IEXT(IS(\texttt{ex:b}))$

$I(G'_9) = false$, i.e., I doesn't satisfy **any** triple in G'_9:

Simple Interpretations 4/4

Dealing with blank nodes is analogously to dealing with existential variables in first-order logic:

We call some function $\mu : B \rightarrow IR$ an assignment.
Given an interpretation I, and an assignment μ, $[I + \mu]$ is defined just like I, except that it uses μ to interpret blank nodes.

- Interpreting non-ground graphs:
  - if G is a non-ground RDF graph then $I(G) = true$ if and only if there exists an assignment μ such that $[I + \mu](G) = true$.

Simple Interpretation – Example non-ground graphs

Same interpretation as before, artificial vocabulary:
$\{\texttt{ex:a}, \texttt{ex:b}, \texttt{ex:c}, \texttt{"whatever"}, \texttt{"whatever"}\hat{}\hat{}\texttt{ex:b}\}$
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G_{10} :

```
_:x <ex:a> <ex:b> .
<ex:c> <ex:b> _:y .
```

$I(G_{10}) = true$, i.e., $I \models G_{10}$:

E.g. take the assignment $\mu(x) = 2, \mu(y) = 1$

Simple Interpretation – Example non-ground graphs

Same interpretation as before, artificial vocabulary:
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G'_{10} :

```
_:x <ex:a> <ex:b> .
<ex:c> <ex:b> _:x .
```

$I(G'_{10}) = false$, i.e., $I \not\models G'_{10}$:

If μ maps x to 1 then the first triple is false, and if it maps it to 2 then the second one.

Simple Entailment between RDF Graphs

The usual entailment relation as we know it from first-order theories:

Simple Entailment

An RDF graph G (simply) entails a graph E, written $G \models E$, if every interpretation which satisfies G also satisfies E

"Entailment is the key idea which connects model-theoretic semantics to real-world applications" [Hayes, 2004] ... indeed, simple entailment is the key for SPARQL graph pattern matching.

Simple Entailment between RDF Graphs

The usual entailment relation as we know it from first-order theories:

Simple Entailment (for sets of graphs)

A set S of RDF graphs (simply) entails a graph E, written $S \models E$, if every interpretation which satisfies **every member of** S also satisfies E

"Entailment is the key idea which connects model-theoretic semantics to real-world applications" [Hayes, 2004] . . . indeed, simple entailment is the key for SPARQL graph pattern matching.

Simple Entailment - Properties

Merging lemma

The merge of a set S of RDF graphs is entailed by S, and entails every member of S, i.e.
$S \models \biguplus_{s \in S} s$ and $\biguplus_{s \in S} s \models s'$, where $s' \in S$.

Simple Entailment - Properties

Merging lemma

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$S \models \biguplus_{s \in S} s$ and $\biguplus_{s \in S} s \models s'$, where $s' \in S$.

Recall the example from before:
G'_{10}:

```
_:x <ex:a> <ex:b> .
<ex:c> <ex:b> _:x .
```

This example shows the difference of union and merge:
The merge of each triple by itself taken as a singleton graph is NOT equivalent to G'_{10}!

(Recall the definition of merge: Obtained by “standardizing apart” blank nodes.)

Simple Entailment - Properties

Main result for simple RDF inference is:

Interpolation Lemma

S entails a graph E if and only if a subgraph of S is an instance of E.

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S entails a graph E if and only if a subgraph of S is an instance of E.

What does this mean?
Recall: We call $\mu(G)$ an *instance* of G, where μ maps blank nodes to UBL.
So, you can test entailment $G \models ?G'$ by

1. guessing a mapping μ and
2. test whether $\mu(G') \subseteq G$

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Complexity

Checking simple entailment is NP-complete.

(proof in the end of the slides, time allowed)

Simple Entailment - Examples 1/4

G_1 :

```
ex:alice foaf:knows ex:bob.
ex:alice foaf:name "Alice".
foaf:knows rdfs:domain foaf:Person.
```

G_3 :

```
_:alice foaf:knows ex:bob.
_:alice foaf:name _:name.
```

G_4 :

```
_:alice foaf:knows ex:bob.
_:alice foaf:name _:alice.
```

Simple Entailment - Examples 1/4

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```

G_3 :

$Alice$ `foaf:knows ex:bob.`
$Alice$ `foaf:name` $Name$.

G_4 :

$Alice$ `foaf:knows ex:bob.`
$Alice$ `foaf:name` $Alice$.

$G_1 \models G_3$:

Simple Entailment - Examples 1/4

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G_3 :

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$Alice$ `foaf:name` $Name$.

G_4 :

$Alice$ `foaf:knows ex:bob.`
$Alice$ `foaf:name` $Alice$.

$G_1 \models G_3$:
$\mu(Alice) = ex:alice, \mu(Name) = "Alice" \Rightarrow \mu(G_3) \subseteq G_1$

Simple Entailment - Examples 1/4

G_1 :

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ex:alice foaf:knows ex:bob.
ex:alice foaf:name "Alice".
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```

G_3 :

$Alice$ foaf:knows ex:bob.
$Alice$ foaf:name $Name$.

G_4 :

$Alice$ foaf:knows ex:bob.
$Alice$ foaf:name $Alice$.

$G_1 \not\models G_4$:

Simple Entailment - Examples 1/4

G_1 :

```
ex:alice foaf:knows ex:bob.
ex:alice foaf:name "Alice".
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$Alice$ foaf:knows ex:bob.
$Alice$ foaf:name $Name$.

G_4 :

$Alice$ foaf:knows ex:bob.
$Alice$ foaf:name $Alice$.

$G_1 \not\models G_4$:
no blank node mapping μ makes $\mu(G_4)$ a subset of G_1

Simple Entailment - Examples 1/4

G_1 :

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Simple Entailment - Examples 1/4

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Simple Entailment - Examples 1/4

G_1 :

```
ex:alice foaf:knows ex:bob.
ex:alice foaf:name "Alice".
foaf:knows rdfs:domain foaf:Person.
```

G_3 :

$Alice$ foaf:knows ex:bob.
$Alice$ foaf:name $Name$.

G_4 :

$Alice$ foaf:knows ex:bob.
$Alice$ foaf:name $Alice$.

$G_4 \models G_3$:

Simple Entailment - Examples 1/4

G_1 :

```
ex:alice foaf:knows ex:bob.
ex:alice foaf:name "Alice".
foaf:knows rdfs:domain foaf:Person.
```

G_3 :

$Alice$ foaf:knows ex:bob.
$Alice$ foaf:name $Name$.

G_4 :

$Alice$ foaf:knows ex:bob.
$Alice$ foaf:name $Alice$.

$G_4 \models G_3$:
$\mu(Alice) = Alice, \mu(Name) = Alice \Rightarrow \mu(G_3) \subseteq G_4$

Simple Entailment - Examples 2/4

G_7 : non-lean

X `foaf:knows ex:bob.`

X `foaf:knows` Y `.`

G_8 : lean

X `foaf:knows ex:bob.`

X `foaf:knows` X `.`

G_7' : lean

X `foaf:knows ex:bob.`

G_8' : lean

X `foaf:knows` X `.`

Simple Entailment - Examples 2/4

G_7 : non-lean

X `foaf:knows ex:bob.`

X `foaf:knows` Y `.`

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X `foaf:knows ex:bob.`

X `foaf:knows` X `.`

G'_7 : lean

X `foaf:knows ex:bob.`

G'_8 : lean

X `foaf:knows` X `.`

$G_7 \not\models G_8$, $G_7 \not\models G'_8$

Simple Entailment - Examples 2/4

G_7 : non-lean

X `foaf:knows ex:bob.`
X `foaf:knows` Y`.`

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X `foaf:knows` X`.`

G_7' : lean

X `foaf:knows ex:bob.`

G_8' : lean

X `foaf:knows` X`.`

$G_7 \not\models G_8$, $G_7 \not\models G_8'$
$G_8 \models G_7$, $G_7 \models G_7'$

Simple Entailment - Examples 2/4

G_7 : non-lean

X `foaf:knows ex:bob.`

X `foaf:knows` Y `.`

G_8 : lean

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X `foaf:knows` X `.`

G'_7 : lean

X `foaf:knows ex:bob.`

G'_8 : lean

X `foaf:knows` X `.`

$G_7 \not\models G_8$, $G_7 \not\models G'_8$

$G_8 \models G_7$, $G_7 \models G'_7$

Finally: $G'_7 \models G_7$!!!! that confirms non-leanness![2]

[2] since G'_7 is a subgraph that is a proper instance entailing the whole graph

Simple Entailment - Examples 3/4

Now what about G_2?

G_1 :

```
ex:alice foaf:knows ex:bob.
ex:alice foaf:name "Alice".
foaf:knows rdfs:domain foaf:Person.
```

G_2 :

```
ex:alice rdf:type foaf:Person.
```

Obviously, no simple entailment: $G_1 \not\models G_2$!

Simple Entailment - Examples 3/4

Now what about G_2?

G_1 :

```
ex:alice foaf:knows ex:bob.
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```

G_2 :

```
ex:alice rdf:type foaf:Person.
```

Obviously, no simple entailment: $G_1 \not\models G_2$!

Would need “special” interpretation of the `rdf:` and `rdfs:` vocabulary!

This is needed to interpret *ontologies*. . .

Recall from before – The FOAF ontology:

- **Properties:** `foaf:name`, `foaf:knows`, `foafhomepage`, `foaf:primaryTopic` etc.
- **Classes:** `foaf:Person`, `foaf:Agent`, `foaf:Document`, `foaf:Organisation`, etc.
- **Relations:** e.g.
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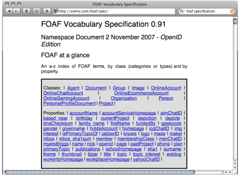

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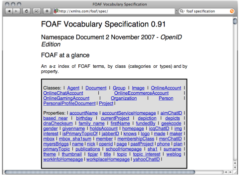

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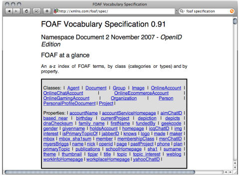

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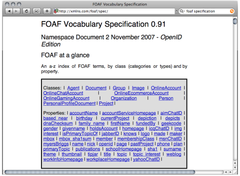

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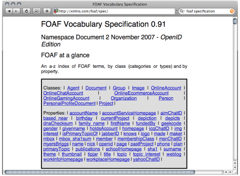
  - *`homepage` denotes **unique** homepage of an `Agent`* (uniquely identifying property)

  ⋮



Simple Entailment - Examples 4/4

G'_1 :

```
ex:alice foaf:knows ex:bob.
ex:alice foaf:name "Alice".  ex:alice ex:age "30.0"^^xs:decimal.
```

G_{FOAF}: <http://xmlns.com/foaf/0.1/>

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foaf:knows rdfs:domain foaf:Person.
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Intuitively, $G'_1 \uplus G_{FOAF}$ should entail: G'_2 :

```
ex:alice rdf:type foaf:Person.
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ex:alice rdf:type foaf:Agent.
ex:bob rdf:type foaf:Agent.
ex:alice ex:age "30"^^xs:integer
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ex:alice rdf:type foaf:Person.      ... because the domain of knows is Person
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ex:bob rdf:type foaf:Agent.	... because each Person is an Agent
ex:alice ex:age "30"^^xs:integer	... simply because each $30.0 = 30$

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- D-entailment: Interpreting datatypes
  - e.g. imposing that in all interpretations that `"1"^^xs:integer` is interpreted the same as `"1.0"^^xs:decimal`

Unit Outline

Simple RDF Entailment is NP-complete: Membership

Recall, we had that before already: We can test entailment $G \models ?G'$ by

1. guessing a mapping μ and
2. test whether $\mu(G') \subseteq G$ (this is obviously polynomial)

Membership in NP - done

Simple RDF Entailment is NP-complete: Hardness

To proof hardness we have to reduce another NP-hard problem to RDF entailment (in polynomial time). Let's "adapt" the proof from [Chandra and Merlin, 1977].

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Reduction (the "trick" is we have to convert an undirected to a directed RDF graph):

- Graph G_1: simply encodes all "allowed" edges:
```
:red :edge :green.   :green :edge :red.
:green :edge :blue.   :blue :edge :green.
:blue :edge :red.   :red :edge :blue.
```
- Graph G_2: for each $(node_1, node_2) \in Gr$ we add two triples:
```
_:n1 :edge _:n2.   _:n2 :edge _:n1.
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  to the graph G_2, i.e, we model the nodes as blank nodes.

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Now, it is easy to see that:

Proposition

Gr is 3-colorably if and only if $G_1 \models G_2$

Recommended Reading

- [Gutiérrez *et al.*, 2004], excellent article on the logical foundations of RDF
- [de Bruijn *et al.*, 2005], relating RDF entailment to normal first-order logic.

A bit more tough reading (specs), but also recommended:

- [Hayes, 2004, Sections 1–2], official RDF semantics specification.
- [Mallea *et al.*, 2011] . . . all you ever wanted to know about blank nodes and never dared to ask.

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