Lógica y Metodos avanzados de Razonamiento - Exercises 1

19 de octubre 2006

Whatever we do not solve during the lecture is homework!

1. First find a reasonable signature (constants, function symbols, predicate symbols) and then write down the following sentences as first order formulas:

   - All humans are mortal.
   - Socrates is a human.
   - There exists human which is immortal.

   Is it possible to find a model for this set of formulas?

2. First find a reasonable signature (constants, function symbols, predicate symbols) and then write down the following sentences as first order formulas:

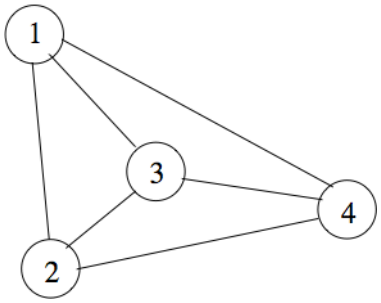
   - Some animals eat meat, others are vegetarian.
   - Animals eating animals are not vegetarian. Grass, Vegetables, Fruits are plants. Vegetarians eat only plants.
   - Cows are animals.
   - Belle is a Cow.
   - Belle eats Grass.
   - Perro is an a Dog.
   - Perro eats Belle.

   Think about the following: After having written this down: is there a model for your set of formulae where Perro is a vegetarian? If yes, what would you have to add to invalidate this model?

3. Write down the following sentences about a graph as first order formulae. Use the binary predicate symbols *edge*, *hasColor* and the unary predicate *node*, and constants $1, 2, 3, \ldots$:

   - Each node has either the color green, red or blue.
   - No two nodes which are connected have the same color.

   Assume you write the following graph down as a conjunction of atomic formulae:



Is it possible to find a model for this set of formulae?

4. Given the following closed First Order formula:

$$\forall x\ \exists y\ gt(y, s(x)) \wedge \forall(a(x, y, z) \rightarrow a(s(x), y, s(z)) \wedge a(x, null, x))$$

   (a) Find a model for this formula, i.e. specify an interpretation $\mathcal{I}$ for the alphabet consisting of the variable symbols x, y, z the predicate symbols $gt/2, a/3$, the constant $null$ and the function symbol $s/1$ which evaluates the formula to $true$.

   (b) Use the evaluation function $Val^{\mathcal{I}}$ to evaluate the truth value of

$$\exists x \exists y(a(s(s(s(0))), s(x), y) \rightarrow gt(y, s(s(s(x)))))$$

   wrt. this interpretation.

5. Write down some unsatisfiable First order sentences

6. Write down some valid First Order sentences

Note that solving these exercises is for your benefit! You can send solutions and questions to me by Monday via e-mail: *axel.polleres@deri.org*